Abstract—This paper considers demodulate-and-forward (DmF)-based incremental relaying (IR) in a distributed setting, where each relay makes the decision on cooperation locally to eliminate network-wide coordination. Unlike traditional IR schemes that use destination feedback merely to activate relays for cooperation, the considered schemes include quantized channel state information (CSI) in destination feedback to facilitate local relay decision. Moreover, two timer-based backoff schemes are employed as the distributed relay selection method. Important performance metrics including the outage probability and the average duration to complete a transmission cycle are derived in closed-form for flat Rayleigh fading channels. Comparisons with the well-known distributed relaying schemes are conducted. Numerical results demonstrate that DmF-based IR with quantized feedback serves as a promising solution for low-complexity power-limited wireless networks.

Index Terms—Demodulate-and-forward (DmF), diversity, delay, incremental relaying, medium access control (MAC), outage probability, quantization.

I. INTRODUCTION

Cooperative relaying has attracted recent research interests for its improvement over the point-to-point transmission [1]–[3]. By processing and forwarding the information overheard from their neighbors, several intermediate nodes in the network form a virtual antenna array to exploit spatial diversity even with single-antenna terminals.

Common relaying approaches such as amplify-and-forward (AF) and decode-and-forward (DF) have been widely studied in the literature. A variant of DF is the demodulate-and-forward (DmF), where relays only demodulate and re-modulate the received signals to ease the decoding overhead. While DmF is attractive for its simplicity, it is vulnerable to error propagation if using traditional diversity techniques such as maximum-ratio-combining (MRC). Recently, a suboptimal demodulator called cooperative-MRC (C-MRC) is proposed in [4], where the combining weight is adapted to the end-to-end channel between the source and the destination via the relay (which we call the two-hop relay channel). It is shown that using C-MRC, DmF achieves full diversity, but its distributed implementation has less been discussed.

Generally, the diversity gain provided by cooperative relaying may come at the cost of degraded spectral efficiency due to information repetition. In the context of single channel systems, two approaches have been proposed to mitigate spectral efficiency loss. Incremental relaying (IR) improves the spectral efficiency by invoking relays to help via the destination feedback only when necessary (i.e., when the source transmission fails) [1], and its performance has been studied in numerous works with the emphasis on the end-to-end error rate. For example, [5] obtained the bit error rate (BER) and the outage probability of AF-based IR in a single-relay network. [6] extended the consideration to the multi-relay network, in which the best relay selection for cooperative relaying and adaptive modulation for variable-rate transmission were considered. The authors in [7] proposed another form of IR, referred to as distributed switch and stay combing (DSSC), where the diversity path is chosen between the direct channel and the two-hop relay channel depending on which has a superior quality. DSSC does not require diversity combiner and thus simplifies the hardware complexity, but the maximum diversity order of this scheme is at most two. Choi et al. [8] also proposed to reduce the combiner complexity by an adaptive IR scheme, which employs a small number of relay nodes in the first round of cooperation. If the destination node still can not recover the source information after the first round, the second round is performed by invoking the rest of the relay nodes. This scheme is shown to achieve full diversity in AF-based IR.

Another countermeasure against spectral efficiency loss is to select either a single or partial relays as the partner. Numerous best-relay selection schemes have been proposed to ensure that full diversity is achieved with the aid of a single best relay. For example, opportunistic relaying (OR) selects the best relay with the strongest bottleneck of the two-hop relay channel [9]. In the context of DF, relays that successfully decode the source signal forms the decoding set in which the one with the strongest relay-destination channel is selected, referred to as selection cooperation (SC) [1]. In terms of multi-relay selection, [10] considered to employ a subset AF relays with
stronger relay-destination channels than the others. For DF, threshold-based selection cooperation (TRSC) is considered in [11], [12], where the selected relays are those with the received SNRs larger than a threshold. [13] further combined IR and best relay selection for DF and characterized the resulting diversity-multiplexing tradeoff.

In this paper, we are interested in DmF-based IR in cooperative networks, considering its low requirements on hardware complexity and simplicity for practical implementation. The main contributions of this paper are summarized as follows.

- For DmF-based IR networks, the scheme in [14], [15] is improved by combining CSI-based random backoff and quantized destination feedback to facilitate distributed relay cooperation. We call such a scheme as selective-IR (S-IR). In addition to the reduced feedback overhead using quantized CSI, numerical results show that S-IR achieves full diversity order.
- Besides the CSI-based random backoff, we also consider a discrete-time random backoff scheme as a variant of the exponential random backoff in IEEE 802.11 protocol. Unlike the CSI-based counterpart that results in a single relay selected, the discrete-time backoff scheme allows multiple relays to transmit in an arbitrary order consecutively. Such a scheme is referred to as randomized-IR (R-IR).
- A comprehensive comparison of various distributed relaying schemes is conducted, including S-IR, R-IR, and two well-known best relay selection schemes: OR and SC. Two important performance metrics, namely, outage probability and average cycle length, are evaluated with closed-form expressions obtained for a multi-relay network over slow flat-Rayleigh fading channel. Outage probability is a key figure of merit in wireless communications, and average cycle length measures how fast a data transmission can be finished via relay cooperation, considering various key delay components.
- Both analytical and simulation results are presented to get insight into different relaying approaches in the context of DmF. In terms of outage performance, S-IR performs as good as OR at low rate, and both outperform SC. At high rate, S-IR provides a little gain over OR and SC. Using the same amount of relays, R-IR performs best at low rate, but it incurs a large performance loss at high rate due to information repetition. As to average delay performance, all the considered schemes perform comparably at low rate except SC. We also show that with a moderate radio turn-around speed, CSI-based random backoff is a superior choice than discrete-time random backoff for distributed relay cooperation. Our results suggest that DmF-based IR using CSI-based random backoff with quantized feedback is a promising solution for low-complexity power-limited distributed wireless networks.

The remainder of this paper is organized as follows. Sec. II introduces the system model. The details of distributed DmF-based IR are described in Sec. III. Performance analysis is conducted in Sec. IV for outage probability and in Sec. V for average cycle length. Sec. VI presents the numerical results and discussions. The paper is concluded in Sec. VII.

II. SYSTEM MODEL

Consider a cooperative network consisting of a source $s$, a destination $d$, and a set of relays $\mathcal{R} = \{r_1, \cdots, r_M\}$. Each node equips a half-duplex radio such that the transmission and reception needs to occur at different time instants. Cooperative relays are altruistic, meaning that they are always willing to assist the source transmission and DmF is performed at relays. The source buffer is always saturated and thus $s$ transmit immediately whenever the previous transmission finishes either by itself or relays. Information bits are encoded into codewords or packets each of which is mapped into blocks consisting multiple symbols and is transmitted with the average power $\mathcal{E}$. We use a general notation, $h_{a,b}$ to denote the channel coefficient of the link between nodes $a$ and $b$, and $|h_{a,b}|$ denotes the corresponding channel gain. Define the instantaneous received SNR as $\gamma_{a,b} = |h_{a,b}|^2/\bar{\gamma}$, with the average SNR (ASN) $\bar{\gamma} := \mathcal{E}/N_0$. The received average SNR due to propagation loss is characterized by $\gamma_{a,b}/(d_{a,b}/d_0)^{-\alpha}$, where $d_{a,b}$ is the distance between nodes $a$ and $b$, $d_0$ is the reference distance, and $\alpha$ is pathloss exponent. All the wireless channels are subject to independent fading, and perfect received CSI is assumed as in [9], [11]-[13], [16]. For diversity combining, the C-MRC proposed in [4] is adopted at $d$.

III. DISTRIBUTED DMF-BASED IR

In this section, we first introduce the principle of DmF-based IR, followed by two timer-based approaches to facilitate its distributed implementation.

A. DmF-based IR

IR works similarly to the typical two-phase cooperative relaying schemes, including the source transmission phase and relay cooperation phase [1]. Differently, IR employs destination feedback to enable the second phase, i.e., relay cooperation is performed only when the source transmission fails. In this case, cooperating relays access the channel according to a certain medium access control (MAC) protocol. In this work, we focus on distributed MAC as will be discussed in Sec. III-B and Sec. III-C.

In IR, destination feedback is used only for activating/deactivating the relays to cooperate and is independent from relay selection. This creates the problem in DmF, where all the relays hearing the request from the destination will be activated. Providing orthogonal channels to a large number of relays may deteriorate the spectral efficiency and introduce extra delay [1]. The problem can be alleviated by selecting a single relay to help as in SC and OR. However, SC and OR permanently employ the selected relay regardless the source transmission result and thus may still suffer spectral efficiency loss.

To enhance existing IR in a distributed setting, [14] proposes to use the CSI of the direct link for relay selection. Specifically, when the retransmission is required, $d$ broadcasts a negative acknowledgement (NACK) message along with the CSI of the
s-d channel. The CSI can be in the form of instantaneous or average SNR measured at the receiving end during source broadcasting. In this work, we consider the use of instantaneous SNR as in [1], [9]. A relay \( r_i \) then compares the SNR of the weakest hop of the two-hop relay channel with that of the s-d channel and participates in relay selection only if \( \min\{\gamma_{s,r_i}, \gamma_{r_i,d}\} \geq \gamma_{s,d} \). The rationale behind this heuristic rule is that, the two-hop relay channel of a DmF relay can be regarded as an equivalent single-hop channel whose SNR denoted as \( \gamma_{eq,i} \) is well approximated by \( \min\{\gamma_{s,r_i}, \gamma_{r_i,d}\} \) (refer to [4] for the details). When \( \gamma_{eq,i} < \gamma_{s,d} \), it suggests that the retransmission by \( r_i \) is not better off than that by \( s \) itself. Consequently, a better candidate than \( s \) can be selected if we exclude the relay with a weaker \( \gamma_{eq,i} \) than \( \gamma_{s,d} \) from the selection process. Another interesting feature of the aforementioned selection rule is that, it adapts to the s-d channel quality such that more relays will be invoked to enhance the transmission reliability when the s-d channel is weaker.

The additional overhead due to CSI feedback can be reduced by using the quantized \( \gamma_{s,d} \) instead of the perfect one. Feedback quantization is an effective technique for reducing feedback load in wireless networks [17], [18]. With \( L \) quantization bits, \( L \) quantization levels are created and each is defined by the lower and upper quantization thresholds. Let \( Q_l, l = 0, \ldots, L - 1 \) be the quantization levels of the SNRs defined by [17]

\[
Q_l = [q_l, q_{l+1}), 0 \leq l \leq L - 1
\]

\[
q_0 = 0, \quad q_L = \infty.
\]  

Using \( L \) quantization bits corresponds to additional overhead of \( \lfloor \log_2 L \rfloor \) bits, where \( \lfloor \cdot \rfloor \) represents the ceiling function. When \( q_l \leq \gamma_{s,d} < q_{l+1} \), the quantization level \( Q_l \) is indicated in the destination feedback. As a result, the relay decision on cooperation can be expressed as

\[
\min(\gamma_{s,r_i}, \gamma_{r_i,d}) \approx \gamma_{eq,i} \geq q_l,
\]  

where \( \gamma_{s,r_i} \) and \( \gamma_{r_i,d} \) can be obtained at relay \( r_i \) by measuring the source broadcast and the destination feedback, respectively. Given \( Q_l \), the set of relays satisfying (2) is referred to as the forwarding set defined by

\[
F_l = \{r_i \in R : \gamma_{eq,i} \geq q_l\}.
\]  

Only relays in the forwarding set will participate in relay selection. When \( F_l \neq \emptyset \), cooperative relaying may be performed by a single best relay or by multiple relays [19]. In the context of IR, we call the former as selective IR (S-IR) and the latter as the randomized IR (R-IR). Both S-IR and R-IR can be implemented with the timer-based backoff scheme, as explained in the sequel.

**B. S-IR**

Motivated by [4], the best relay is selected as the one with the highest \( \gamma_{eq,i} \) in the forwarding set. The selection can be performed in a distributed fashion using the CSI-based backoff timer as in [9]. After receiving the destination feedback containing \( Q_l \), a relay \( r_i \in F_l \) sets a backoff timer to \( \delta_l = \lambda/\gamma_{eq,i} \), where \( \lambda \) is a timing parameter relevant to the collision probability. Based on the backoff-timer \( \delta_i \), \( r_i \) counts down the timer to zero and transmits. Since \( \delta_i \) is reciprocal to \( \gamma_{eq,i} \), the relay with the highest \( \gamma_{eq,i} \) has its timer expired first. Notice that the collision of two or more relays may happen if their timers expire within the same timer interval, whose length is dominated by how fast the transceiver of the half-duplex radio switches between its operating mode, referred to as the turn-around time. As suggested in [9], given the hardware specification and the desired collision probability, one can determine the proper value for \( \lambda \). As such, S-IR selects the best relay via CSI-based backoff timer, and each source transmission is assisted by at most one relay when necessary.

**C. R-IR**

To allow multiple relays to help in a distributed manner, we introduce R-IR as a simple extension of traditional automatic repeat and request (ARQ) protocols in the sense that the source data is retransmitted by relays until it is received successfully or no more relays can be used [19]. After receiving the NACK from \( d \), each relay maintains a random backoff timer uniformly selected from \([0, CW - 1]\), where \( CW \) is the contention window size. The relay counts down the timer slot by slot until it reaches zero and then transmits in the succeeding time slot. The slotted time resolution can be provided by the timing synchronization function as in IEEE 802.11 WLANs. Without using any prioritized mechanism and CSI, relay transmission order in R-IR is arbitrary.

Unlike S-IR that adopts a continuous-time backoff timer, the timer in R-IR operates in a discrete-time basis. Whenever two or more relays select the same timer value, their simultaneous transmissions will cause collisions. Extensive collisions might happen if node population is large and the contention window size is small. Choosing a large contention window helps to reduce collisions at the cost of the prolonged delay. Alternatively, collisions can be reduced by deactivating the relay whose transmission does not yield a successful combining. In details, if \( d \) fails to decode after a relay transmission, it indicates in the NACK whether the failure is caused by collisions or unsuccessful diversity combining. For the former case, the transmitted relay can keep contending the channel by resetting its backoff timer and increasing the retry counter by one. For the latter case, the transmitted relay suspends from cooperation. However, if all the relays have transmitted and \( d \) still fails to decode, the channel will be persistently idle. To avoid unbounded delay, the destination can set a maximum delay timer, whose length depends on the delay tolerance of the underlying traffic. The maximum backoff timer is triggered whenever the NACK is delivered. If the channel remains idle when this timer expires, the current transmission should be terminated immediately.

**IV. OUTAGE PERFORMANCE**

This section analyzes the outage performance of DmF-based S-IR and R-IR by deriving compact equations for the outage probability. An outage event occurs when the instantaneous mutual information falls below a threshold for a target spectral
efficiency denoted as R in bps/Hz [6]. For comparisons, two well-known distributed relaying schemes, namely OR and SC are also included in the analysis, considering their similarity with S-IR in selecting the best relay for cooperation.

For simplicity, all the two-hop relay channels experience independent but statistically identical fading with the same mean pathloss [16]. However, the average received SNR of the s-d channel is not necessarily equal to that of the equivalent two-hop relay channel. For a random variable (RV) X, the following notations are used throughout the analysis: $f_X(x)$, $F_X(x)$, and $\bar{X}$ represent the probability density function (PDF), the cumulative distribution function (CDF), and the expected value, respectively.

Under the assumption of long-term quasi-static fading, a data block experiences an identical channel realization during each transmission round [20]. The mutual information due to direct transmission between s and d is

$$I_0 = \log_2(1 + \gamma_{s,d}).$$

(4)

If cooperative relaying is invoked, diversity combining the source and m relay transmissions forms an equivalent channel between s and d with the mutual information

$$I_m = \frac{1}{m+1} \log_2 \left( 1 + (\gamma_{s,d} + \gamma_{eq,i}) \right),$$

(5)

where $\gamma_{eq,i}$ is the equivalent SNR of the two-hop relay channel associated with relay $r_i$ based on DmF.

A. OR

For a fair comparison, the DF-based OR scheme proposed in [9] is modified as follows: the best relay is defined in the same way as in S-IR, i.e., the one with the highest $\gamma_{eq,i}$. For $|\mathcal{R}| = M$, denote the instantaneous SNR of the two-hop relay channel associated with the best relay as $\gamma_{eq}^*$, i.e., $\gamma_{eq,M}^* = \max\{\gamma_{eq,i}\} |r_i \in \mathcal{R}\}$. The mutual information resulted from combining the transmissions of the source and the best relay is [13], [21]

$$I^{OR} = \frac{1}{2} \log_2 \left( 1 + (\gamma_{s,d} + \gamma_{eq,M}) \right).$$

(6)

Using [13, Appendix I], the outage probability can be obtained as

$$P_{out}^{OR} = \Pr[I^{OR} < R] = \frac{1}{M} \left( \frac{M}{2} \right)^{M+1} i \Psi_1(i, b_2, \bar{\gamma}_{s,d}, \bar{\gamma}_{eq}),$$

(7)

where $\bar{\gamma}_{eq} = \bar{\gamma}_{eq,1} = \cdots = \bar{\gamma}_{eq,M}$ and

$$\Psi_1(i, b_2, x, y) =
\begin{cases}
\frac{1}{ix - y} \left( x(1 - e^{-b_2}) - y(1 - e^{-i\frac{b_2}{\bar{\gamma}_{s,d}}}) \right), & \text{if } x \neq y, \\
\frac{y}{x} - e^{-b_2} \left( \frac{y}{x} + \frac{1}{\bar{\gamma}_{s,d}} \right), & \text{if } x = y,
\end{cases}$$

with $b_2 = 2^{2R} - 1$.

B. SC

In SC, the best relay is chosen as the one with the strongest r-d channel in the decoding set defined as $\mathcal{D} = \{r_i \in \mathcal{R}: \gamma_{s,r_i,d} \geq b_2\}$ [22]. Denote $\gamma_{r,d}^*$ as the SNR of the r-d channel associated with the best relay in the decoding set, i.e., $\gamma_{r,d}^* = \max\{\gamma_{r,d}|r_i \in \mathcal{D}\}$. According to [22], the outage probability is given by

$$P_{out}^{SC} = P_{out}^{dir} \cdot \Pr[|\mathcal{D}| = 0] + \sum_{m=1}^{M} \frac{M}{m} \cdot \Pr[|\mathcal{D}| = m].$$

(10)

In (10), $P_{out}^{dir}$ accounts for the case when $\mathcal{D} = \emptyset$ where the mutual information of the s-d channel is identical to 1/2$I_0$. Thus,

$$P_{out}^{dir} = \Pr[\frac{1}{2} l_0 < R | |\mathcal{D}| = 0] = 1 - e^{-\frac{b_2}{\bar{\gamma}_{s,d}}}.\tag{11}$$

The conditional outage probability $P_{out,|\mathcal{D}|=m}$ given $m \in [1, M]$ relays in the decoding set is equal to $\Pr[|\mathcal{D}| = m], \Pr[|\mathcal{D}| = m], \Pr[|\mathcal{D}| = m], \Pr[|\mathcal{D}| = m]$, where the mutual information

$$I_{m}^{SC} = \frac{1}{2} \log_2 \left( 1 + (\gamma_{s,d} + \gamma_{r,d}) \right), \ \ |\mathcal{D}| = m.\tag{12}$$

Hence, the outage probability of SC can be derived in the same manner as (7), leading to

$$P_{out,|\mathcal{D}|=m} = \sum_{i=1}^{m} \left( i \right)(-1)^{i+1} i \Psi_1(i, b_2, \bar{\gamma}_{s,d}, \bar{\gamma}_{r,d}).$$

(13)

Finally, the size of the decoding set has the PDF [1]

$$P_{out,|\mathcal{D}|=m} = \left( \frac{M}{m} \right) e^{-\frac{b_2}{\bar{\gamma}_{s,d}}} \left( 1 - e^{-\frac{b_2}{\bar{\gamma}_{s,d}}} \right) M - m,\tag{14}$$

for $m \in [0, M]$. Substituting (11), (13), and (14) into (10), the outage probability of SC is obtained as

$$P_{out}^{SC} = \left( 1 - e^{-\frac{b_2}{\bar{\gamma}_{s,d}}} \right) \left( 1 - e^{-\frac{b_2}{\bar{\gamma}_{r,d}}} \right) M + \sum_{m=1}^{M} \left( \frac{M}{m} \right) e^{-\frac{b_2}{\bar{\gamma}_{r,d}}} \times \left( 1 - e^{-\frac{b_2}{\bar{\gamma}_{r,d}}} \right) M - m,$$

(15)

C. S-IR

In S-IR, relays are invoked if the source transmission fails. Additionally, the best relay is selected from the forwarding set $\mathcal{F}_l$, as defined in (3), given the quantized feedback $Q_l$. Accordingly, the overall outage probability is

$$P_{out}^{S-IR} = \sum_{l=0}^{L-1} \left[ P_{out}^{dir} \cdot \Pr[|\mathcal{F}_l| = 0] \right] + \sum_{m=1}^{M} \left[ P_{out,|\mathcal{F}_l|=m} \cdot \Pr[|\mathcal{F}_l| = m] \right] \cdot \Pr[Q_l].\tag{16}$$

In (16), $Pr[Q_l]$ represents the probability of receiving feedback $Q_l$, which can be derived as [See Appendix A]

$$Pr[Q_l] = \begin{cases} e^{-\frac{q_l}{\bar{\gamma}_{s,d}}} - e^{-\frac{q_{l+1}}{\bar{\gamma}_{s,d}}}, & 0 \leq q_l, q_{l+1} < b_1, \\
e^{-\frac{q_l}{\bar{\gamma}_{s,d}}}, & 0 \leq q_l < b_1, q_{l+1} \geq b_1, \\
0, & \text{otherwise.}
\end{cases}$$

(17)
Similar to (14), the PDF of $F_l$ is given as
\[
\Pr[|F_l| = m] = \binom{M}{m} e^{-\frac{M}{\gamma_{eq}}} (1 - e^{-\frac{m}{\gamma_{eq}}})^{M-m}.
\] (18)
If $|F_l| = 0$, only $s$ transmits and the corresponding outage probability is given by (11). For $|F_l| = m > 0$, the conditional outage probability with the aid of the best relay is defined as
\[
P_{\text{out}, |F_l| = m} = \Pr\left[ (l_0 < R) \cap (|F_l| = m < R) \right].
\] (19)
In (19), $I_{m}^{\text{IR}}$ denotes the mutual information of the equivalent channel between $s$ and $d$ given by
\[
I_{m}^{\text{IR}} = \frac{1}{2} \log_2 (1 + \gamma_{s,d} + \gamma_{eq,m}),
\] (20)
where $\gamma_{eq,m}$ represents the end-to-end SNR associated with the best relay. Following some derivation [see Appendix B], (19) can be obtained as
\[
P_{\text{out}, |F_l| = m} = \sum_{i=1}^{m} \binom{m}{i} \left[ \frac{-1}{\gamma_{sd}} e^{-\frac{(b_i-x)_{i}}{\gamma_{eq}}} \Psi_2(i, b_i, \gamma_{s,d}, \gamma_{eq}), \right.
\] (21)
where $b_1 = 2^k - 1$, and
\[
Psi_2(i, b_0, x, y) = \left\{ \begin{array}{ll}
1 - e^{-\frac{1}{x}} - \frac{x}{y} b_0, & y \neq ix,
\frac{1-e^{-\frac{1}{x}} - \frac{x}{y} b_0}{x}, & y = ix.
\end{array} \right.
\] (22)
Substituting (11), (17), (18), (21) into (16), we obtain a closed-form expression for the outage probability of S-IR, which is omitted due to space limit.

D. R-IR
In R-IR, relays transmit one-by-one in an arbitrary order, utilizing the time diversity as in the conventional ARQ protocol. With the aid of $m$ relays, the outage probability is
\[
P_{\text{out}, m}^{\text{R-IR}} = \Pr\left[ (l_0 < R) \cap (l_m < R) \right],
\] (23)
Following the approach in Appendix B, we obtain
\[
P_{\text{out}, m}^{\text{R-IR}} = 1 - e^{-\frac{b}{\gamma_{eq}}} - e^{-\frac{b_{m+1}}{\gamma_{eq}}} \Psi_3(m, b_1, b_{m+1}, \gamma_{s,d}, \gamma_{eq}),
\] (24)
where $b_{m+1} = (2^{(m+1)k} - 1)$ and
\[
\Psi_3(m, a, b, x, y) = \left\{ \begin{array}{ll}
\sum_{i=0}^{m} \sum_{k=0}^{i} \frac{\gamma_{k+1}^{k+1} b_k}{i! x^i y^k} - (k+1) \Psi_2(m-k+1, \gamma_{eq}, b_k, \gamma_{s,d}), & x \neq y,
\sum_{i=0}^{m} \sum_{k=0}^{i} \frac{i! b_k^{i-k}}{x^i y^{i-k}}, & x = y.
\end{array} \right.
\] (25)

V. AVERAGE CYCLE LENGTH
In this section, we analyze the average cycle length for distributed relaying schemes based on time orthogonality. Assuming a saturated source queue, we refer to the duration between two consecutive block transmissions as the cycle length. Specifically, we consider numerous delay components within a transmission cycle, including the time for transmitting a block denoted as $t_a$, the time for destination feedback denoted as $t_f$, and that for relay selection denoted as $t_s$.

A. AVERAGE DATA TRANSMISSION TIME
For OR and SC, a transmission cycle always contains two data transmissions from $s$ the selected relay, respectively. Thus the time for data transmissions per cycle in these two schemes is constant, namely, $t_d^{\text{OR}} = t_d^{\text{SC}} = 2 \cdot t_{data}$, where $t_{data}$ is the block transmission time. For S-IR, the selected relay is invoked only when the source transmission fails. Thus the average time for data transmissions per cycle is
\[
\bar{t}_d^{\text{S-IR}} = (1 \cdot \Pr[l_0 \geq R] + 2 \cdot \Pr[l_0 < R]) \cdot t_{data}
\] (26)
For R-IR, the number of data transmissions in a cycle equals $1 + m$, where $m$ is a discrete random variable in $[0, M]$ representing the number of transmitted relays per cycle. As a result, the average data transmission time is
\[
\bar{t}_d^{\text{R-IR}} = \sum_{m=0}^{M} (1 + m) \cdot \Pr[m] \cdot t_{data}.
\] (27)
The probability of having $m$ relays transmission per cycle is given by (20), (5)
\[
\Pr[m] = \Pr[(l_0 < R) \cap \cdots \cap (l_m \geq R)]
\] (28)
which involves $m$-fold integrals and thus does not yield compact expressions. In (20), it is shown that $\Pr[m]$ can be determined by the diversity combining results of the $(m-1)$th and the $m$th rounds of cooperative relaying when the random sequence $(l_i)_{i=0}^{m}$ is nondecreasing and $(l_i < R) \subseteq (l_i < R)$ for $i < j$. In R-IR, however, $(l_i)_{i=0}^{m}$ may not be a nondecreasing random sequence because of information repetition as can be seen from (5). For analytical tractability, however, we resort to the memoryless property as follows: if cooperation relaying is required, the event that a transmission is successful with the aid of $m$ relays only depends on the diversity combining results of the $m$th round of cooperative relaying. This implies $\Pr[m] \approx \Pr[l_m \geq R]$, which can be found in (29) at the top of this page, where $b_{m+1} = 2^{(m+1)k} - 1$, and $\Psi_3(\cdot, \cdot, \cdot, \cdot, \cdot)$ is given in (25).

B. AVERAGE FEEDBACK TIME
Both OR and SC are feedback-free schemes and thus incur zero feedback time, i.e., $t_f^{\text{OR}} = t_f^{\text{SC}} = 0$. For S-IR, $d$ always echoes source transmission with an ACK/NACK, resulting in $t_f^{\text{S-IR}} = t_{ack} + t_q$, where $t_{ack}$ represents the time for transmitting ACK/NACK and $t_q$ accounts for additional time for transmitting quantized $\gamma_{s,d}$. For R-IR, the number of required ACK/NACK signaling is identical to the number of data transmissions in a transmission cycle. Similar to (27), the average time for feedback transmission per cycle in R-IR is
\[
\bar{t}_f^{\text{R-IR}} = \sum_{m=0}^{M} (1 + m) \cdot \Pr[m] \cdot t_{ack}.
\] (30)

C. AVERAGE RELAY SELECTION TIME
The time to select the cooperating relays in distributed networks is primarily determined by the underlying channel
access mechanism and also which set of relays that participate in relay selection. We consider two classes of distributed DmF schemes: the best relay selection scheme with CSI-based backoff including OR, SC, and S-IR, and the multi-relay cooperation scheme with discrete random backoff, i.e., R-IR.

1) OR: In OR, all the relay nodes participate in relay selection and the time to select the best relay is equal to

$$t_{s, OR} = \min_{\delta_i \in \mathcal{D}} \delta_i = \lambda/\gamma_{eq,i}$$  \hspace{1cm} (31)

According to the definition of $\gamma_{eq,i}$, it is an exponentially distributed random variable with the mean given by $\overline{\gamma}_{eq,i} = \tilde{\gamma}_{eq,i} = \tilde{\gamma}_{eq,r,d}/(\tilde{\gamma}_{eq,r,d} + \tilde{\gamma}_{eq},d)$. Given $|\mathcal{D}| = M$, the maximum of $M$ i.i.d. $\gamma_{eq,i}$'s has the mean given by

$$\overline{\gamma}_{eq,M} = \frac{\overline{\gamma}_{eq}}{M} \sum_{m=0}^{M} (M-m)^{-1}.$$  \hspace{1cm} (32)

As a result, the average relay selection time in OR is

$$t_{s, OR} = \frac{\lambda}{\overline{\gamma}_{eq} \sum_{m=0}^{M} (M-m)^{-1}}.$$  \hspace{1cm} (33)

2) SC: In this scheme, only the relays in the decoding set $\mathcal{D}$ participate in relay selection. If $\mathcal{D} = \emptyset$, the delay equals to the maximum delay timer value $\chi$. Otherwise, the backoff timer associated with the best relay in the decoding set determines the selection time, i.e.,

$$t_{s, SC} = \begin{cases} \min_{\delta_i \in \mathcal{D}} \delta_i, & \mathcal{D} \neq \emptyset, \\ \chi, & \mathcal{D} = \emptyset. \end{cases}$$ \hspace{1cm} (34)

Accordingly, the average of $t_{s, SC}$ can be obtained with the aid of (14) and (32) as given by

$$t_{s, SC} = \chi(1 - \frac{b_{0}}{\overline{\gamma}_{eq}})M + \sum_{m=1}^{M} \frac{\lambda(M-m)e^{-\overline{\gamma}_{eq}(1 - \frac{b_{0}}{\overline{\gamma}_{eq}})(m-1)}}{m}.$$  \hspace{1cm} (35)

3) S-IR: According to (2), only the relays in the forwarding set $\mathcal{F}_l$ participate in relay selection. Therefore, the relay selection time is a function of $\mathcal{F}_l$ given as

$$t_{s, S-IR} = \begin{cases} \min_{\delta_i \in \mathcal{F}_l} \delta_i, & \mathcal{F}_l \neq \emptyset, \\ \chi, & \mathcal{F}_l = \emptyset. \end{cases}$$ \hspace{1cm} (36)

and its mean value can be found using (18) and (36)

$$t_{s, S-IR} = \sum_{l=0}^{L-1} \left[ (1 - \frac{b_{0}}{\overline{\gamma}_{eq}})M \cdot \chi \right. + \sum_{m=1}^{M} \frac{\lambda(M-m)e^{-\overline{\gamma}_{eq}(1 - \frac{b_{0}}{\overline{\gamma}_{eq}})(m-1)}}{m} \left. \right] \cdot \Pr[Q_l],$$ \hspace{1cm} (37)

where $\Pr[Q_l]$ and $\overline{\gamma}_{eq,m}$ are given in (17) and (32), respectively.

4) R-IR: In R-IR, a relay is said to be selected if it earns the channel access by contents. Due to potential collisions, each selection may experience multiple rounds of contents. Denote $\omega_m$ the contention period associated with $m$ contending relays. If $m = 1$, the average contention period is simply $(1 + CW)/2$. For $2 \leq m \leq M$, $\omega_m$ is identical to the minimum of $m$ i.i.d. discrete uniform RVs with the mean value given by [23]

$$\overline{b}_m = \sum_{i=1}^{CW} i \cdot \left[ (1 - \frac{i}{CW})^m - (1 - \frac{i}{CW})^m \right], \hspace{1cm} m \in [2, M].$$ \hspace{1cm} (38)

Notice that if more than one relay chooses the minimum backoff counter, the collision due to $m$ contending relays occurs with probability [23]

$$P_{col,m} = 1 - \frac{m\sum_{i=1}^{CW-1}(CW-i)^{m-1}}{CW^m}.$$ \hspace{1cm} (39)

Since each contending relay has the equal chance to earn the channel access, the number of contentions rounds within a transmission cycle follows a geometric distribution and thus the average contention period is $\overline{\omega}_m = \overline{b}_m/(1 - P_{col,m})$, assuming infinite retry limit. The special case $m = 0$ implies no more relays can cooperate and the wireless medium will be idle until the maximum delay timer $\chi$ expires, and hence $\overline{\omega}_0 = \chi$. Summarizing the above results we have

$$\overline{\omega}_m = \begin{cases} \chi, & m = 0 \\ \frac{1 + CW}{2}, & m = 1 \\ \frac{1}{m \sum_{i=1}^{CW-1}(CW-i)^{m-1}}, & m = 2, \ldots, M. \end{cases} \hspace{1cm} (40)$$

If the selected relay cannot yield a successful decoding at $d$, additional rounds of relay selection take place before the termination of a transmission cycle. Denote the number of relay selection in a transmission cycle as $n'$ and consider the following cases.

- A successful direct transmission, and thus zero relay selection time.
- Successful decoding after $n' \in [1, M]$ rounds of relay selection. Since a relay that has gained channel access successfully becomes deactivated, $n'$ decreases by one between two consecutive contention periods. Therefore, the total relay selection time in this case is $\sum_{n=M-(n'-1)}^{M} \overline{\omega}_n$.
- Unsuccessful decoding yet all relays have been used. In this case, the relay selection process will experience $M$ contention periods plus a duration equal to the maximum delay timer because no more relays are active.
Based on the above discussion, the average relay selection time per cycle can be computed as

$$t_{s}^{R-IR} = \sum_{m'=1}^{M} \Pr[m'] \cdot \sum_{n=M-(m'-1)}^{M} \omega_{n} + \Pr[M] \cdot \chi,$$

where $\Pr[m']$ is given in (29), and $\Pr[M]$ accounts for the case that the current cycle ends with outage after using all the relays as given by

$$\Pr[M] = \Pr[(l_{0} < R) \cap (l_{M} < R)] = 1 - e^{-\frac{b_{0}}{\gamma_{s,d}}} - e^{-\frac{b_{M}}{\gamma_{eq}}} \Psi_{3}(M, b_{1}, b_{M+1}, \gamma_{s,d}, \gamma_{eq}).$$

VI. NUMERICAL RESULTS

Numerical results are presented in this section to assess the performance of various DmF-based distributed relaying schemes, including the three best relay selection schemes OR, SC, and S-IR, and the multi-relay cooperation scheme R-IR. Simulations are also performed to validate the analytical results. For the network considered, the optimal relay position for minimizing the end-to-end error rate (or maximizing the equivalent end-to-end SNR) is in the middle between $s$ and $d$. Accordingly, we set $d_{s,r} = d_{r,d} = 0.5d_{s,d}$, where $d_{s,d} = 2 \cdot d_{0} = 20$ m and the pathloss exponent $\alpha = 3$. This node topology will be used throughout this section. For convenience, all the timing parameters are normalized to the data transmission time, which is subject to the data block size and the physical transmission rate. Unless specified, the normalized turn-around time of the half-duplex radio is equal to 0.2, the normalized maximum delay timer $\chi$ has the value of 2, the normalized ACK/NACK transmission time is 0.2, and the contention window $CW$ size used in R-IR equals 8. The quantization thresholds used in S-IR are determined using the method in [18] for the multi-level quadrature amplitude modulation (M-QAM) with $L = 4$, corresponding to a two-bit overhead.

We first show the outage probability versus ASNR $\gamma$ for low rate ($R = 0.7$ bps/Hz) in Fig. 1(a) and high rate ($R = 2$ bps/Hz) in Fig. 1(b), respectively, with $M = 3$. For presentation brevity, only analytical results are plotted, except for S-IR that also includes the simulated ones with $M = 1, 2$. Among the three best relay selection schemes, S-IR achieves comparable performance as OR and it slightly outperforms SC at low rate. As outage rate raises to 2 bps/Hz, S-IR provides a little gain over OR and SC, as shown in Fig. 1(b). The improvement is a direct consequence of using the destination feedback, by which the relays are invoked only when the direct transmission fails to achieve a better channel utilization. On the other hand, OR always performs better than SC, and this is contrary to the result reported in [22]. The reason is that the OR scheme considered in [22] selects the best relay from the decoding set, i.e., $\gamma_{s,r} \geq b_{2}$, which is imposed to avoid error propagation by DF relays. Consequently, it is possible to have no relay to help, resulting in the relay phase being idle. In the DmF-based OR considered here, there is always one relay selected to help that improves the diversity combiner output. Concerning R-IR, it performs best at high ASNR and low outage rate (in Fig. 1(a)), while it becomes the worst at high outage rate (in Fig. 1(b)), indicating the pronounced spectral efficiency loss. From both figures, the analytical results agree with the simulated ones validating the accuracy of our analysis, and S-IR achieves full diversity as OR and SC.

We show the impact of quantization resolution to the outage performance of S-IR in Fig. 2, where the quantization level varies from 2 to 64, corresponding to the overhead of 1 to 6 bits. The result of using perfect $\gamma_{s,d}$ is also shown for comparison. As can be seen, even a one-bit quantization feedback works sufficiently well, which justifies the practice of introducing quantized feedback to facilitate distributed cooperation decision making.

Next, we evaluate the average cycle length of the best relay schemes. We first show the results of OR in Fig. 3 with $M = 5, 10$, and $R = 2$ bps/Hz. It is depicted that a better channel quality and more participating relays reduce the average cycle length, which can be explained as follows. Because OR maintains a constant data transmission time and

![Figure 1](attachment:image1.png)

(a) $R = 0.7$ bps/Hz.

![Figure 2](attachment:image2.png)

(b) $R = 2$ bps/Hz, $M = 3$. 

Fig. 1. Average outage probability versus ASNR ($\gamma$) with varied number of relays ($M$) and outage rate ($R$).
zero feedback time, we can focus on the impact of average relay selection time \( t^R_{\text{OR}} \) in (37). Assuming \( \gamma_{s,r} = \gamma_{r,d} = \gamma \), (37) is reduced to \( t^R_{\text{OR}} = \lambda/(\gamma \cdot \sum_{m=0}^{M} (M - m)^{-1}) \), which is inversely proportional to \( \gamma \) and \( M \). Furthermore, the delay performance is more relevant to channel quality than the number of participating relays. Compared the analytical results with the simulated ones, a certain discrepancy between them exists at low ASNR (\(< 10 \) dB) and they match closely at high ASNR (\( \geq 10 \) dB). The inaccuracy at low ASNR is due to the fairly large variance of the cycle length as shall be seen next. For SC and S-IR, we observe similar results and thus they are omitted.

Fig. 4(a) and Fig. 4(b) compare the first and the second order statistics of the normalized average cycle length of all the considered schemes. Here we fix \( M = 10 \) and \( R = 0.7 \) bps/Hz, and vary ASNR. Only simulation results obtained from \( 10^5 \) independent runs are shown, except R-IR with analytical ones included in Fig. 4(a). All the considered schemes can converge to the minimum consumed delay as ASNR improves. At low SNR (i.e., \( 0 \sim 5 \) dB), SC incurs a relatively higher average cycle length. To explain this, we recognize the fact that both the data transmission time and the feedback time in SC are constant. It is thus sufficient to examine the average relay selection time \( t_s \) only. As mentioned above, the average relay selection time of the best relay selection scheme is inversely proportional to the ASNR and the number of participating relays. Because of the more stringent constraint on the decoding set in SC, the number of participating relays (or equivalently, the size of decoding set) is always less than that in OR (i.e., the total number of relays) and it tends to be less than that in S-IR (i.e., the size of forwarding set). Consequently, SC performs worst than OR and S-IR, particularly when ASNR is low that results in a larger backoff timer of the selected (best) relay. Moreover, the smaller number of the participating relays also leads to a larger variation of the random backoff timer value, as depicted in Fig. 4(b), which shows the variance of the normalized cycle length. The results indicate that the
delay variance of OR, SC, and S-IR is fairly large at low SNR, because the random backoff counter used in these schemes may range from zero to infinity. On the contrary, R-IR uses a discrete backoff counter bounded by the maximum contention window size $CW$, a tunable design parameter subject to the delay tolerance and node population.

![Graph showing normalized average cycle length of R-IR with $M = 10$.](image)

Fig. 5. Normalized average cycle length of R-IR with $M = 10$.

For R-IR using discrete random backoff, the delay performance is affected by the contention window size $CW$ as can be seen from (40) and also Fig. 5. Among the three $CW$ values (4, 8, and 32) considered, the minimum delay performance appears at $CW = 8$. This observation differs from that found in classical contention-based MAC protocols such as carrier sense multiple access with collision avoidance (CSMA/CA) used in WLANs, where a contention window size larger than the node population is favored in terms of delay reduction. In R-IR, the relay nodes deactivate themselves after accessing the channel successfully and hence the contention intensity reduces as a transmission cycle proceeds. A rule of thumb would be to select a contention window size close to but smaller than the number of relays.

Finally, we illustrate the delay performance of the distributed cooperative schemes using CSI-based backoff timer versus the radio turn-around time in Fig. 6 for $M = 10$, $R = 2$ bps/Hz, and $\bar{\gamma} = 10, 20$ dB. The result of R-IR with $CW = 8$ is also shown for comparison. As the turn-around time increases, the average cycle length linearly increases for all the schemes using CSI-based backoff timer. The effect of turn-around time is minor at high ASNR (e.g., 20 dB), while it significantly affects the average cycle length at lower ASNR (e.g., 10 dB). Compared with R-IR using discrete random backoff, the normalized turn-around time exceeding 0.6 and 1.2 at $\bar{\gamma} = 20$ dB and $\bar{\gamma} = 10$ dB, respectively, will cause the delay performance of OR and S-IR worse than R-IR. Similar to the result in Fig. 4(a), SC performs poor at lower ASNR and high outage rate. The results shown here indicate that S-IR achieves the best delay performance with less requirement on hardware capability (namely, the speed of radio turn-around) than other relaying schemes based on distributed best relay selection.

![Graph showing average cycle length versus normalized radio turn-around time with $M = 10$ and $R = 2$ bps/Hz.](image)

Fig. 6. Average cycle length versus normalized radio turn-around time with $M = 10$ and $R = 2$ bps/Hz.

### VII. Conclusions

To facilitate distributed implementation of DmF-based incremental relaying, this paper considered the problem of allowing relay nodes to locally decide whether to access the channel for cooperative relaying. Since DmF is performed, all the relay nodes in the network are eligible to forward. This may lead to an intense contention among the relay nodes and in turn the unfavorable delay. To mitigate the contention intensity, the destination node feeds back the information of the source-destination channel as a means to limit the contendng relay nodes to a reasonably smaller population. For the sake of performance evaluation, closed-form expressions for the average outage probability and the average cycle length are derived. Our results show that with distributed channel access, DmF-based incremental relaying can still achieve full diversity as it does in the centralized channel access, and the average cycle length can be effectively reduced using limited feedback from the destination. While this paper specifically considers the communication between one source-destination pair, more general scenarios such as multicast [24] and multihop [25] in distributed cooperative networks should deserve further research.

### Appendix A

$Pr[Q_l]$ in (17)

Recall that the quantization level $Q_l$ associated with $\gamma_{s,d}$ is chosen when the $s$-$d$ channel is in outage and its SNR is between the quantization thresholds $q_l$ and $q_{l+1}$. From (4), outage occurs on $s$-$d$ channel when the mutual information $l_0$ is less than the outage rate $R$, or equivalently, $\gamma_{s,d} < b_0$, where $b_0 := 2^R - 1$. Thus the effective SNR of the $s$-$d$ channel seen at the diversity combiner is a RV define as

$$\gamma'_{s,d} = \begin{cases} \gamma_{s,d}, & 0 \leq \gamma_{s,d} < b_0, \\ 0, & \gamma_{s,d} \geq b_0. \end{cases}$$  

(A.1)
Meanwhile, the probability that $\gamma_{s,d}$ is mapped to a particular quantization level $Q_l$ is

$$\Pr[Q_l] = \Pr[q_l \leq \gamma_{s,d} < q_{l+1}] = F_{\gamma_{s,d}}(q_{l+1}) - F_{\gamma_{s,d}}(q_l). \quad (A.2)$$

To find $F_{\gamma_{s,d}}(q_{l+1})$ and $F_{\gamma_{s,d}}(q_l)$, we start from the PDF based on (A.1) as given by

$$f_{\gamma_{s,d}}(\gamma') = \begin{cases} (1 - F_{\gamma_{s,d}}(b_0)) \delta(\gamma'), & \gamma' = b_0, \\ 0, & b_0 < \gamma' < \infty, \\ F_{\gamma_{s,d}}(\gamma'), & 0 \leq \gamma' < b_0. \end{cases} \quad (A.3)$$

Then the CDF can be obtained as

$$F_{\gamma_{s,d}}(\gamma') = \int_0^{\gamma'} f_{\gamma_{s,d}}(x) dx = \begin{cases} F_{\gamma_{s,d}}(\gamma'), & 0 \leq \gamma' < b_0, \\ 1, & \gamma' \geq b_0. \end{cases} \quad (A.4)$$

Since $\gamma_{s,d}$ is exponentially distributed with mean $\bar{\gamma}_{s,d}$, (A.2) can be readily obtained based on (A.4) considering the following cases. Specifically, if $0 \leq q_l, q_{l+1} < b_0$, $\Pr[Q_l] = e^{-q_l/\bar{\gamma}_{s,d}} - e^{-q_{l+1}/\bar{\gamma}_{s,d}}$. For $0 \leq q_l < b_0$ and $q_{l+1} \geq b_0$, $\Pr[Q_l] = e^{-q_l/\bar{\gamma}_{s,d}}$. Otherwise, $\Pr[Q_l] = 0$.

### APPENDIX B

**CONDITIONAL OUTAGE PROBABILITY OF S-IR IN (19)**

From (19) and (20), the conditional outage probability $P_{out,|F_1|=m}$ can be written as

$$P_{out,|F_1|=m} = \Pr \left[ (\gamma_{s,d} < b_0) \cap (\gamma_{s,d} + \gamma_{eq,m} < b_2) \right]. \quad (B.5)$$

For Rayleigh fading, $\gamma_{s,d}$ is exponentially distributed and $\gamma_{eq,m} = \max\{\gamma_{eq,i}|r_i \in F_1, |F_1| = m\}$ is the maximum of $m$ i.i.d. clipped exponential random variable conditioned on $|F_1| = m$, i.e, $\gamma_{eq,m} = \max\{\gamma_{eq,i}|i = 1\}$ with

$$\gamma_{eq,i} = \begin{cases} \gamma_{eq,i}, & \gamma_{eq,i} \geq q_l, \\ 0, & 0 \leq \gamma_{eq,i} < q_l. \end{cases} \quad (B.6)$$

The following lemma gives closed-form expressions for (19).

**Lemma 1:** Consider an exponential random variable $X$ with mean $\bar{X}$, and $m$ i.i.d. clipped exponential random variables $Y_i$, all with the value bounded from below by a nonnegative constant $q_l$ and the same mean $\bar{Y}$. Define the variable $Y^* = \max\{Y_i\}_{i=1}^m$. Given two nonnegative constants $a$ and $b$ with $a < b$, the joint probability that $X$ is less than $a$ and $(X + Y^*)$ is less than $b$ is given by

$$\Pr [ (X < a) \cap (X + Y^* < b) ] = \begin{cases} \sum_{i=0}^{m} \binom{m}{i} \frac{(-1)^i}{X} e^{-\frac{(b-q_l)}{X}} \Psi_2(i, a, \bar{X}, \bar{Y}), & a \leq b - q_l, \\ 0, & \text{otherwise}, \end{cases}$$

where

$$\Psi_2(i, a, \bar{X}, \bar{Y}) = \begin{cases} 1 - e^{-\frac{\bar{X}}{\bar{Y}}} - i a, & \bar{Y} \neq i \bar{X}, \\ a, & \bar{Y} = i \bar{X}. \end{cases} \quad (B.7)$$

### REFERENCES


