Direct Detection and Identification of WIMP Dark Matter

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Direct Dark Matter detection

Overview
Direct detection phenomenology
Direct detection techniques
Backgrounds and background discriminations
Directional detection experiments

Model-independent WIMP identification

Motivation
Reconstruction of the 1-D WIMP velocity distribution
Determination of the WIMP mass
Estimation of the SI scalar WIMP-nucleon coupling
Determinations of ratios of WIMP-nucleon cross sections

Summary
Evidence for Dark Matter
Evidence for Dark Matter

- Clusters of galaxies (1930s)

[https://insidetheperimeter.ca/what-we-know-and-what-we-dont-about-dark-matter/]

[Coma cluster of galaxies; F. Zwicky (1933)]
Evidence for Dark Matter

- Rotation curves of spiral galaxies (1970s)

[https://insidetheperimeter.ca/what-we-know-and-what-we-dont-about-dark-matter/]

Evidence for Dark Matter

- Collision of two clusters of galaxies (Bullet Cluster, 1E 0657-56)


[http://chandra.harvard.edu/photo/2006/1e0657/]
Evidence for Dark Matter

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Evidence for Dark Matter

- Collision of two clusters of galaxies (Bullet Cluster, 1E 0657-56)

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Evidence for Dark Matter

- Anisotropy of the cosmic microwave background radiation (CMBR)

[NASA/WMAP Science Team, WMAP 5-year result (2008)]
Evidence for Dark Matter

- Abundances of the light elements: D, $^3$He, $^4$He, $^7$Li

[Review of Particle Physics 2016, 24. Big-Bang Nucleosynthesis]
Evidence for Dark Matter

- Supernovae type Ia (SNe Ia) at high-redshift

[Supernova Cosmology Project 2010]
Evidence for Dark Matter

- Astronomical measurements
  - Cosmic microwave background (CMB)
  - Anisotropy of the CMB radiation (CMBR)
  - Present expansion rate of the Universe, Hubble constant
  - Age of the Universe
  - Abundances of the light elements: D, $^3$He, $^4$He, $^7$Li
  - Opacity of the Lyman-\(\alpha\) forest toward high-redshift quasars
  - Gas-to-total mass ratio
  - Mass-to-light ratio
  - Peculiar velocities of galaxies
  - Shape of the present power spectrum of density perturbations
  - Supernovae type Ia (SNe Ia) at high-redshift
Evidence for Dark Matter

- A large fraction of the mass/energy in our Universe is dark!

  - Dark Energy: $\Omega_\Lambda = 0.692 \pm 0.012^\dagger$
  - (Cold) Dark Matter: $\Omega_{\text{CDM}} = 0.258 \pm 0.011^\dagger$
  - Baryonic matter: $\Omega_b = 0.0484 \pm 0.0010^\dagger$
  - Luminous matter: $\Omega_{\text{lum}} \sim (3.5398 \pm 0.0470) \times 10^{-5}^\ddagger$
  - Stars: $0.002 \lesssim \Omega_* \lesssim 0.003^\S$
  - Neutrinos: $0.0012 \leq \Omega_\nu < 0.016^\dagger$
  - CMB photons: $\Omega_\gamma = 5.38(15) \times 10^{-5}^\dagger$

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Candidates for Dark Matter
Candidates for Dark Matter

Particles of the Standard Model

## Candidates for Dark Matter

### Particles of the Standard Model

<table>
<thead>
<tr>
<th>SM particles</th>
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<tbody>
<tr>
<td><strong>Name</strong></td>
<td><strong>Symbol</strong></td>
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<tr>
<td>up-quarks</td>
<td>$u, c, t$</td>
</tr>
<tr>
<td>down-quarks</td>
<td>$d, s, b$</td>
</tr>
<tr>
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<td>$e, \mu, \tau$</td>
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<tr>
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<td>$\nu_e, \nu_\mu, \nu_\tau$</td>
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<td>photon</td>
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<tr>
<td>Higgs boson</td>
<td>$H$</td>
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<tr>
<td>W bosons</td>
<td>$W^\pm$</td>
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</tbody>
</table>
# Candidates for Dark Matter

Particles of typical **supersymmetric** models

<table>
<thead>
<tr>
<th>Normal particles</th>
<th>SUSY partners</th>
</tr>
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<tr>
<td>light scalar Higgs</td>
<td>$h^0$</td>
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<tr>
<td>heavy scalar Higgs</td>
<td>$H^0$</td>
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<tr>
<td>pseudoscalar Higgs</td>
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<td>charged Higgs</td>
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<tr>
<td>W bosons</td>
<td>$W^\pm$</td>
</tr>
<tr>
<td>graviton</td>
<td>G</td>
</tr>
<tr>
<td>axion</td>
<td>a</td>
</tr>
</tbody>
</table>

Weakly Interacting Massive Particles (WIMPs)
Candidates for Dark Matter

- **Cold Dark Matter (CDM)**
  - moved non-relativistically when galaxies could just start to form (matter-radiation decoupling time).
  - would form some small galactic scale structures due to their relatively slower velocities (bottom-up).

- **Hot Dark Matter (HDM)**
  - moved relativistically at the matter-radiation decoupling time.
  - would cover great(er) distances and form some very large scale structures (top-down).

- **Warm Dark Matter (WDM)**

- **Dark baryons**
Candidates for Dark Matter

- Weakly Interacting Massive Particles (WIMPs) $\chi$
  - Dark Matter relic density
    \[ \Omega_{\text{CDM}} h^2 = 0.1186 \pm 0.0020^\dagger \sim \frac{0.1 \text{ pb} \cdot c}{\langle \sigma_{\text{anni}} v \rangle} \]
    \[ [^\dagger: \text{Review of Particle Physics 2016, 2. Astrophysical Constants and Parameters}] \]
    - $\sigma_{\text{anni}} \sim 10^3$ pb
    - Weak interaction
      - Gravitational, EM, strong interactions
    - Mass ranges roughly between 10 GeV and a few TeV.
      - $v_{\text{CDM}} \sim 10^{-3} c \Rightarrow Q \sim \left( \frac{10^{-6}}{2} \right) m_\chi c^2$

- Gravitino, axion, axino
Candidates for Dark Matter

- The lightest neutralino \( \tilde{\chi}_1^0 \)
  - The lightest SUSY particle (LSP)
  - Linear combinations of photino, Z-ino and neutral higgsinos

- The lightest Kaluza-Klein (KK) particle (LKP) in Universal Extra Dimensions (UED) models

- The lightest T-odd particle in Little Higgs (LH) models

- Other candidates:
Part I:
Direct Dark Matter detection
Dark Matter searches
Dark Matter searches

DM should have **small, but non-zero** interactions with SM matter.

→ Three different ways to detect DM particles

- **Colliders**
  - $(p, e) → p, e^+$
  - $(p, e) → \text{DM, DM}$

- **Indirect detection**
  - $(\text{DM, DM}) → e^+, \bar{p}, \bar{D}$
  - $(\text{DM, DM}) → e, \nu_\mu, \gamma$

- **Direct detection**
  - $(\text{DM, DM}) → q$
Direct Dark Matter detection
Direct Dark Matter detection

- Direct Dark Matter detection: elastic WIMP-nucleus scattering
  - The event rate depends on
    - the WIMP density near the Earth $\rho_0$
    - the WIMP-nucleus cross sections $\sigma^{\text{SI}}_0$ and $\sigma^{\text{SD}}_0$
    - the WIMP mass $m_\chi$
    - the velocity distribution of incident WIMPs $f_1(v)$
  - The WIMP-nucleus cross section is about $10^{-3} \sim 10^{-6}$ pb
    - the optimistic expected event rate is $\sim 0.1 \text{ events}/(100 \text{ kg–100 day})$
    - but could be $< 1 \text{ event/ton-yr}$
  - An exponential-like recoil energy spectrum
    - Most events would be with energies less than a few tens keV
  - Typical background events due to cosmic rays and ambient radioactivity: signals $\approx \mathcal{O}(10^6) : 1 \Rightarrow 1 : 1.x$
Direct Dark Matter detection

- Direct Dark Matter detection **phenomenology**
  - (One-dimensional) velocity distribution function $f_1(v)$ or $f(v)$
    - The Solar orbital speed around the Galactic center $v_0$
    - Escape and 1-D maximal cut-off velocities $v_{\text{esc}}$ and $v_{\text{max}}$
    - The Earth’s motion in the Galactic frame
  - Local density $\rho_0$ or halo density distribution $\rho(r)$
  - WIMP mass $m_\chi$
  - WIMP-nucleon (proton/neutron) couplings
    - SI scalar couplings $f_{(p,n)}$
    - SI vector couplings $b_{(p,n)}$
    - SD axial-vector couplings $a_{(p,n)}$
Direct Dark Matter detection

- Direct Dark Matter detection phenomenology
  - (WIMP-nucleus) scattering form factors $F_{SI}^2(Q)$ and $F_{SD}^2(Q)$
    - For nuclei with $A \gtrsim 30$, the SI scalar interaction ($\propto A^2$) would almost always dominate over the SD interaction (in SUSY models).
    - In e.g. UED models, the SI and SD interactions are comparable.
  - Minimal and (kinematic) maximal cut-off energies $Q_{(\text{min, max})}$, $Q_{\text{max, kin}}$
  - Detector materials
    - Ge ⇔ Si
    - Xe ⇔ Ar
    - I ⇔ F
Direct Dark Matter detection

- Direct Dark Matter detection phenomenology
  - Developing data analysis procedures
  - Comparison of different models on the $\sigma_{\chi(p/n)}^{\text{SD}}$ vs. $\sigma_{\chi(p/n)}^{\text{SI}}$ planes
  - Constraining/distinguishing particle/astronomical halo models
  - Reconstructing WIMP properties
  - Combining with and being complementarity of indirect Dark Matter detection and collider experiments
  - Packages for Monte Carlo simulations and/or (real) data analyses
Direct Dark Matter detection

- Time-dependence of the velocity distribution/event rate
  - Annual modulation of the event rate

  ![Diagram showing Earth and Sun orbits](image)

  - Due to the orbital motion of the Earth around the Sun
  - A cosinusoidal function with a one-year period, a peak around June 2nd, and an amplitude of $\sim 5\%$ (recoil-energy dependent)
  - The peak could be shifted by $\sim 3$ weeks!

  
  ![Reference]
  
  - [K. Freese, M. Lisanti and C. Savage, Rev. Mod. Phys. 85, 1561 (2013)]
  
  - The nuclear recoil (signal) identification should be performed!

  
  ![Reference]
  
  
Direct Dark Matter detection

- Time-dependence of the velocity distribution/event rate
  - Annual modulation of the event rate

- Results of the DAMA/NaI and DAMA/LIBRA experiments

[R. Bernabei et al., Nucl. Instrum. Meth. A742, 177 (2014)]

[K. Freese, M. Lisanti and C. Savage, Rev. Mod. Phys. 85, 1561 (2013)]
Direct Dark Matter detection

- Time-dependence of the velocity distribution/event rate
  - Diurnal modulation of the event rate
    - Due to the rotation of the Earth
    - Directionality of the WIMP wind
    - Shielding of the incident WIMP flux by the Earth

[C. L. Shan (XAO-CAS)]

http://www.tir.tw/conf/cygnus2017/
Direct Dark Matter detection experiments

- Direct Dark Matter detection techniques
  - Ionization (charges)
  - Scintillation (light)
  - Heat (phonons)
  - Quenching factor
    - Nuclear recoil relative efficiency
    - Measured (electron equivalent) recoil energy $keV_{ee}$
      $\iff$ true nuclear recoil energy $keV_r$
  - Raw/total mass/exposure
    $\iff$ fiducial mass/exposure
- Combinations of two signals
  - Event-by-event background discrimination
  - Down to 5 to 10 keV recoil energy
Direct Dark Matter detection experiments

- **Semiconductor/scintillator detectors**
  - **ANAIS**
    Nal(Tl), Laboratorio Subterráneo de Canfranc (LSC), Spain.
  - **CDEX**
    Ge, China Jin-Ping Laboratory (CJPL), China.
  - **CDMS → SuperCDMS**
    Ge and Si, Soudan Underground Laboratory, Minnesota, USA; Sudbury Neutrino Observatory (SNOLAB), Canada.
  - **CoGeNT → C-4**
    Ge, Soudan Underground Laboratory, Minnesota, USA.
  - **CRESST → EURECA**
    Al₂O₃/CaWO₄, Laboratori Nazionali del Gran Sasso (LNGS), Italy.
  - **DAMA/Nal → DAMA/LIBRA**
    Nal(Tl), Laboratori Nazionali del Gran Sasso (LNGS), Italy.
  - **DM-Ice**
    Nal(Tl), South Pole.
  - **EDELWEISS (EDW) → EURECA**
    Ge, Laboratoire Souterrain de Modane (LSM), France.
  - **KIMS**
    CsI(Tl), Yangyang Laboratory (Y2L), South Korea.
  - **NaIAD**
    Nal(Tl), Boulby Underground Laboratory, UK.
  - **SABRE**
    Nal(Tl), Stawell Underground Physics Laboratory (SUPL), Australia.
Direct Dark Matter detection experiments

- Semiconductor/scintillator detectors
  - CDMS, CRESST and EDELWEISS detectors


Direct Dark Matter detection experiments

- **Liquid noble gas detectors**
  - **ArDM**
    - Dual-phase Ar, Laboratorio Subterráneo de Canfranc (LSC), Spain.
  - **DarkSide**
    - Dual-phase Ar, Laboratori Nazionali del Gran Sasso (LNGS), Italy.
  - **DARWIN**
    - Dual-phase Ar and Xe, Laboratori Nazionali del Gran Sasso (LNGS), Italy
  - **DEAP/CLEAN**
    - Single-phase Ar and Ne, Sudbury Neutrino Observatory (SNOLAB), Canada.
  - **LUX → LZ**
    - Dual-phase Xe, Homestake, USA.
  - **MAX**
    - Dual-phase Ar and Xe, USA.
  - **PandaX**
    - Dual-phase Xe, China Jin-Ping Laboratory (CJPL), China.
  - **WARP**
    - Dual-phase Ar, Laboratori Nazionali del Gran Sasso (LNGS), Italy.
  - **XENON**
    - Dual-phase Xe, Laboratori Nazionali del Gran Sasso (LNGS), Italy.
  - **XMASS**
    - Single-phase Xe, Kamioka, Japan.
  - **ZEPLIN → LZ**
    - Single-/dual-phase Xe, Boulby Underground Laboratory, UK.
Liquid noble gas detectors

- XENON and XMASS detectors

Direct Dark Matter detection experiments

- **Superheated droplet/gas** detectors (with *directional* sensitivity)
  - **COUPP → PICO**
    CF$_3$I, C$_3$F$_8$, and C$_4$F$_{10}$, Sudbury Neutrino Observatory (SNOLAB), Canada.
  - **PICASSO → PICO**
    C$_4$F$_{10}$, Sudbury Neutrino Observatory (SNOLAB), Canada.
  - **SIMPLE**
    C$_2$ClF$_5$ and CF$_3$I, Laboratoire Souterrainé Bas Bruit (LSBB), France.
  - **TREX-DM**
    Ar and Ne, Laboratorio Subterráneo de Canfranc (LSC), Spain.
  - **D3 → CYGNUS**
    Hawaii, USA
  - **DMTPC**
    CF$_4$, Sudbury Neutrino Observatory (SNOLAB), Canada.
  - **DRIFT → CYGNUS**
    73% CS$_2$ + 25% CF$_4$ + 2% O$_2$, Boulby Underground Laboratory, UK.
  - **MIMAC**
    70% CF$_4$ + 28% CHF$_3$ + 2% C$_4$H$_{10}$, Laboratoire Souterrain de Modane (LSM), France.
  - **NEWAGE → CYGNUS**
    CF$_4$, Kamioka, Japan.
  - **NEWSdm**
    Nuclear emulsion, Laboratoire Souterrain de Modane (LSM), France.
Direct Dark Matter detection experiments

- **Superheated droplet/gas detectors** (with **directional** sensitivity)
  - COUPP detector (bubble chamber)

[COUPP Collab., E. Behnke et al., Science 319, 933 (2008)]
Direct Dark Matter detection experiments

- Backgrounds and background discriminations
  - Cosmic muons
  - External natural radioactivity
  - Internal natural radioactivity
  - Fast neutrons
  - Multiple-scatter events
  - Electron recoils
  - Surface events
  - Incomplete charge collection
  - Neutron-induced nuclear recoils
Direct Dark Matter detection experiments

- **Cosmic muons**
  - Induce fast neutrons
  - $\mathcal{O}(10^{10})$ cosmic muons/m$^2$ Earth’s surface/yr
  - Go deep underground (reduced by a factor of $10^5$ to $10^8$)

Direct Dark Matter detection experiments

- **External natural radioactivity**
  - Radioactive isotopes in the rock/walls
  - Passive shielding:
    - high-Z materials (e.g., lead) for MeV $\gamma$-ray,
    - low-Z materials for $\alpha$-, $\beta$-, and low energy $\gamma$-rays

- **Internal natural radioactivity**
  - Radioactive isotopes contamination in the outer shielding, equipment around the detector, and detector material
  - Radiopure materials

- **Fast neutrons**
  - Induced by cosmic-ray in the inner lead shielding
  - Water tank or polyethylene (PE)
    - (materials with high density of hydrogen)
Direct Dark Matter detection experiments

- Multiple-scatter events
  - Mean free path of WIMP-induced events $\sim O$ (light year)
  - Array of detectors
  - CDMS ZIP-detector cryostat and tower

[http://cdms.berkeley.edu/; P. Cushman, JPCS 39, 63 (2006)]
Direct Dark Matter detection experiments

- **Electron recoils**
  - Ionization yield
  - Ionization (S2)/primary scintillation (S1)
  - CDMS-II calibration

[CDMS Collab., Z. Ahmed et al., Science 327, 1619 (2010)]
Direct Dark Matter detection experiments

- Surface events/incomplete charge collection
  - Rising time of phonon pulses
  - Self-shielding
  - CDMS-II calibration

[CDMS Collab., Z. Ahmed et al., Science 327, 1619 (2010)]
Direct Dark Matter detection experiments

- Surface events/incomplete charge collection
  - Rising time of phonon pulses
  - Self-shielding
  - XENON10 result

[XENON10 Collab., J. Angle et al., PRL 100, 021303 (2008)]
Direct Detection and Identification of WIMP Dark Matter

Direct Dark Matter detection

- Backgrounds and background discriminations

Direct Dark Matter detection experiments

- Neutron-induced nuclear recoils
  - Mimic WIMP-induced nuclear recoils
  - Scintillating reflector
  - CRESST-II calibration

[CREST Collab., R. F. Lang et al., Astropart. Phys. 33, 60 (2010)]
Directional Dark Matter detection experiments

- Requirements
  - Recoil track (3D) reconstruction
  - Sense (head-tail) recognition

- Techniques
  - (Low pressure) gaseous time-projection chamber (TPC)
  - Micromegas
  - Gas electron multiplier (GEM)
  - Nuclear emulsion

- Materials
  - CF$_4$, C$_3$F$_8$, C$_4$F$_{10}$
  - CF$_3$I, CHF$_3$, C$_2$ClF$_5$
  - CS$_2$, CH$_4$
  - SF$_6$
Directional Dark Matter detection experiments

- Working principle
  - MIMAC $\mu$TPC detector

![Diagram of MIMAC $\mu$TPC detector](image-url)

[Q. Riffard et al., J. Inst. 12, P06021 (2017)]
Directional Dark Matter detection experiments

- Working principle
  - MIMAC $\mu$TPC detector

[Q. Riffard et al., J. Inst. 11, P08011 (2016)]
Directional Dark Matter detection experiments

- Working principle
  - MIMAC $\mu$TPC detector

![Diagram of MIMAC $\mu$TPC detector](image)

Scheme of a MIMAC $\mu$TPC  
Evolution of the collected charges on the anode

[Q. Riffard et al., J. Inst. 11, P08011 (2016)]
Part II:
Model-independent WIMP identification
Motivation

- Differential event rate for elastic WIMP-nucleus scattering

\[ \frac{dR}{dQ} = A F^2(Q) \int_{v_{\text{min}}(Q)}^{v_{\text{max}}} \left[ \frac{f_1(v)}{v} \right] dv \]

Here

\[ v_{\text{min}}(Q) = \alpha \sqrt{Q} \]

is the minimal incoming velocity of incident WIMPs that can deposit the recoil energy \( Q \) in the detector,

\[ A \equiv \frac{\rho_0 \sigma_0}{2 m_\chi m_{r,N}^2} \quad \text{and} \quad \alpha \equiv \sqrt{\frac{m_N}{2 m_{r,N}^2}} \]

\( \rho_0 \): WIMP density near the Earth

\( \sigma_0 \): total cross section ignoring the form factor suppression

\( F(Q) \): elastic nuclear form factor

\( f_1(v) \): one-dimensional velocity distribution of halo WIMPs

\( m_{r,N} = \frac{m_\chi m_N}{m_\chi + m_N} \)
Motivation

- Differential event rate for elastic WIMP-nucleus scattering

\[
\frac{dR}{dQ} = A F^2(Q) \int_{v_{\text{min}}(Q)}^{v_{\text{max}}} \left[ \frac{f_1(v)}{v} \right] dv
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\(\rho_0\): WIMP density near the Earth

\(\sigma_0\): total cross section ignoring the form factor suppression

\(F(Q)\): elastic nuclear form factor

\(f_1(v)\): one-dimensional velocity distribution of halo WIMPs
Reconstruction of the 1-D WIMP velocity distribution
Reconstruction of the 1-D WIMP velocity distribution

- Normalized one-dimensional WIMP velocity distribution function

\[
f_1(v) = \mathcal{N} \left\{-2Q \cdot \frac{d}{dQ} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right]\right\}_{Q=\nu^2/\alpha^2}
\]

\[
\mathcal{N} = \frac{2}{\alpha} \left\{ \int_0^\infty \frac{1}{\sqrt{Q}} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] dQ \right\}^{-1}
\]

- Moments of the velocity distribution function

\[
\langle v^n \rangle = \mathcal{N}(Q_{\min}) \left( \frac{\alpha^{n+1}}{2} \right) \left[ \frac{2Q_{\min}^{(n+1)/2}}{F^2(Q_{\min})} \left( \frac{dR}{dQ} \right) \right]_{Q=Q_{\min}} + (n + 1)I_n(Q_{\min})
\]

\[
\mathcal{N}(Q_{\min}) = \frac{2}{\alpha} \left[ \frac{2Q_{\min}^{1/2}}{F^2(Q_{\min})} \left( \frac{dR}{dQ} \right) \right]_{Q=Q_{\min}}^{-1} + I_0(Q_{\min})
\]

\[
I_n(Q_{\min}) = \int_{Q_{\min}}^\infty Q^{(n-1)/2} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] dQ
\]

[M. Drees and CLS, JCAP 0706, 011 (2007)]
Reconstruction of the 1-D WIMP velocity distribution

- **Ansatz:** the measured recoil spectrum in the \( n \)th \( Q \)-bin

\[
\frac{dR}{dQ}_{\text{expt}, \, Q \approx Q_n} \equiv r_n e^{k_n(Q - Q_{s,n})}
\]

\[
r_n \equiv \frac{N_n}{b_n}
\]

- Logarithmic slope and shifted point in the \( n \)th \( Q \)-bin

\[
Q - Q_n |_{n} \equiv \frac{1}{N_n} \sum_{i=1}^{N_n} (Q_{n,i} - Q_n) = \left( \frac{b_n}{2} \right) \coth \left( \frac{b_n k_n}{2} \right) - \frac{1}{k_n}
\]

\[
Q_{s,n} = Q_n + \frac{1}{k_n} \ln \left[ \frac{\sinh(b_n k_n/2)}{b_n k_n/2} \right]
\]

- Reconstructing the one-dimensional WIMP velocity distribution

\[
f_1(v_{s,n}) = \mathcal{N} \left[ \frac{2Q_{s,n} r_n}{F^2(Q_{s,n})} \right] \left[ \frac{d}{dQ} \ln F^2(Q) \right]_{Q = Q_{s,n} - k_n}
\]

\[
\mathcal{N} = \frac{2}{\alpha} \left[ \sum_{a} \frac{1}{\sqrt{Q_a} F^2(Q_a)} \right]^{-1}
\]

\[
v_{s,n} = \alpha \sqrt{Q_{s,n}}
\]

[M. Drees and CLS, JCAP 0706, 011 (2007)]
Reconstruction of the 1-D WIMP velocity distribution

- Reconstructed $f_{1,\text{rec}}(v_s,n)$
  
  $^{76}\text{Ge}$, 500 events, $m_\chi = 100$ GeV

\[ \chi^2/\text{dof} = 0.73 \]

[From M. Drees and CLS, JCAP 0706, 011 (2007)]
Reconstruction of the 1-D WIMP velocity distribution

- Reconstructed $f_{1,\text{rec}}(v_s, n)$
  - $^{76}\text{Ge}$, 2 - 50 keV, 500 events, $m_\chi = 25$ GeV

![Graph showing the reconstruction of $f_{1,\text{rec}}(v_s, n)$](attachment:image.jpg)

$^{76}\text{Ge}$, $Q_{\min} > 2$ keV, $Q_{\max} < 50$ keV, 500 events, $m_\chi = 25$ GeV, 4 bins, up to 2 bins per window

[CLS, IJMPD 24, 1550090 (2015)]
Reconstruction of the 1-D WIMP velocity distribution

- **Modification of the renormalization constant**

\[
\mathcal{N} = \frac{2}{\alpha} \left\{ \frac{2 Q_{\text{min}}^{1/2} r(Q_{\text{min}})}{F^2(Q_{\text{min}})} \left[ K_1(Q_{\text{min}}) Q_{\text{min}} + 1 \right] + I_0(Q_{\text{min}}, Q_{\text{max}}^*) \right\}^{-1}
\]

where

\[
r(Q_{\text{min}}) \equiv \left( \frac{dR}{dQ} \right)_{\text{expt}, Q=Q_{\text{min}}} = r_1 e^{k_1(Q_{\text{min}} - Q_s, 1)}
\]

\[
K_n(Q) \equiv \frac{d}{dQ} \left[ \ln F^2(Q) \right] - k_n
\]

\[
I_n(Q_{\text{min}}, Q_{\text{max}}^*) = \int_{Q_{\text{min}}}^{Q_{\text{max}}^*} Q^{(n-1)/2} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] dQ \rightarrow \sum_a \frac{Q_a^{(n-1)/2}}{F^2(Q_a)}
\]

\[
Q_{\text{max}}^* \equiv \min \left( Q_{\text{max}}, Q_{\text{max, kin}} = \frac{v_{\text{max}}^2}{\alpha^2} \right)
\]

[CLS, IJMPD 24, 1550090 (2015); Y. Bai, W. Sun and CLS (2018)]
Reconstruction of the 1-D WIMP velocity distribution

- Reconstructed $f_{1,\text{rec}}(v_s, n)$
  
  $(^{76}\text{Ge}, \text{2.5 - 50 keV}, b_1 = 2.5 \text{ keV}, \text{500 events, } m_\chi = 20 \text{ GeV})$

[C. L. Shan (XAO-CAS)]

[NKU, December 13, 2017]
Reconstruction of the 1-D WIMP velocity distribution

- Reconstructed $f_{1,\text{rec}}(v_s, n)$

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[CLS, IJMPD 24, 1550090 (2015); Y. Bai, W. Sun and CLS (2018)]
Reconstruction of the 1-D WIMP velocity distribution

- Reconstructed $f_{1,\text{rec}}(v_s, n)$
  $\left(^{28}\text{Si}, 2.5 - 50 \ \text{keV}, \ b_1 = 5 \ \text{keV}, \ 500 \ \text{events}, \ m_\chi = 20 \ \text{GeV}\right)$

[CLS, IJMPD 24, 1550090 (2015); Y. Bai, W. Sun and CLS (2018)]
Reconstruction of the 1-D WIMP velocity distribution

- Reconstructed $f_{1,\text{rec}}(v_s, n)$
  
  $(^{28}\text{Si}, 5 - 50 \text{ keV}, b_1 = 5 \text{ keV}, 500 \text{ events, } m_\chi = 20 \text{ GeV})$

[CLS, IJMPD 24, 1550090 (2015); Y. Bai, W. Sun and CLS (2018)]
Reconstruction of the 1-D WIMP velocity distribution

- Theoretical bias estimate of \[ \frac{\Delta_0^{v_{\text{min}}^*} - \int_{0}^{v_{\text{min}}^*} f_1(v) \, dv}{\int_{0}^{v_{\text{max}}^*} f_1(v) \, dv} \]

[CLS, IJMPD 24, 1550090 (2015)]
Bayesian reconstruction of the 1-D WIMP velocity distribution

- Bayesian analysis
  \[ p(\Theta | \text{data}) = \frac{p(\text{data} | \Theta)}{p(\text{data})} \cdot p(\Theta) \]

  - \( \Theta: \{ a_1, a_2, \ldots, a_{N_{\text{Bayesian}}} \} \), a specified (combination of the) value(s) of the fitting parameter(s)

  - \( p(\Theta) \): prior probability, our degree of belief about \( \Theta \) being the true value(s) of fitting parameter(s), often given in form of the (multiplication of the) probability distribution(s) of the fitting parameter(s)

  - \( p(\text{data} | \Theta) \): the probability of the observed result, once the specified (combination of the) value(s) of the fitting parameter(s) happens, usually be described by the “likelihood” function of \( \Theta \), \( L(\Theta) \).

  - \( p(\text{data}) \): evidence, the total probability of obtaining the particular set of data

  - \( p(\Theta | \text{data}) \): posterior probability density function for \( \Theta \), the probability of that the specified (combination of the) value(s) of the fitting parameter(s) happens, given the observed result
Bayesian reconstruction of the 1-D WIMP velocity distribution

- Probability distribution functions for $p(\Theta)$
  - Without prior knowledge about the fitting parameter
    - Flat-distributed
      \[ p_i(a_i) = 1 \quad \text{for} \quad a_{i,\text{min}} \leq a_i \leq a_{i,\text{max}} \]
  - With prior knowledge about the fitting parameter
    - Around a theoretical predicted/estimated or experimental measured value $\mu_{a,i}$
    - With (statistical) uncertainties $\sigma_{a,i}$
    - Gaussian-distributed
      \[ p_i(a_i; \mu_{a,i}, \sigma_{a,i}) = \frac{1}{\sqrt{2\pi} \sigma_{a,i}} e^{-\frac{(a_i - \mu_{a,i})^2}{2\sigma_{a,i}^2}} \]
Bayesian reconstruction of the 1-D WIMP velocity distribution

- Likelihood function for $p(\text{data}|\Theta)$
  - Theoretical one-dimensional WIMP velocity distribution function:
    \[ f_{1,\text{th}}(v; a_1, a_2, \cdots, a_{N_{\text{Bayesian}}}) \]
  - Assuming that the reconstructed data points are Gaussian-distributed around the theoretical predictions

\[
\mathcal{L}
\left(f_{1,\text{rec}}(v_s,\mu), \mu = 1, 2, \cdots, W; a_i, i = 1, 2, \cdots, N_{\text{Bayesian}}\right)
\equiv \prod_{\mu=1}^{W} \text{Gau}
\left(v_s,\mu, f_{1,\text{rec}}(v_s,\mu), \sigma_{f_1,s,\mu}; a_1, a_2, \cdots, a_{N_{\text{Bayesian}}}\right)
\]

with

\[
\text{Gau}
\left(v_s,\mu, f_{1,\text{rec}}(v_s,\mu), \sigma_{f_1,s,\mu}; a_1, a_2, \cdots, a_{N_{\text{Bayesian}}}\right)
\equiv \frac{1}{\sqrt{2\pi} \sigma_{f_1,s,\mu}} e^{- \left[ f_{1,\text{rec}}(v_s,\mu) - f_{1,\text{th}}(v_s,\mu; a_1, a_2, \cdots, a_{N_{\text{Bayesian}}}) \right]^2 / 2\sigma_{f_1,s,\mu}^2}
\]

[CLS, JCAP 1408, 009 (2014)]
Bayesian reconstruction of the 1-D WIMP velocity distribution

- Input and fitting one-dimensional WIMP velocity distribution functions
  - “One-parameter” shifted Maxwellian velocity distribution
    \[ f_{1,\text{sh},v_0}(v) = \frac{1}{\sqrt{\pi}} \left( \frac{v}{v_0 v_e} \right) \left[ e^{-\frac{(v-v_e)^2}{v_0^2}} - e^{-\frac{(v+v_e)^2}{v_0^2}} \right] \quad v_e = 1.05 \, v_0 \]
  - Shifted Maxwellian velocity distribution
    \[ f_{1,\text{sh}}(v) = \frac{1}{\sqrt{\pi}} \left( \frac{v}{v_0 v_e} \right) \left[ e^{-\frac{(v-v_e)^2}{v_0^2}} - e^{-\frac{(v+v_e)^2}{v_0^2}} \right] \]
  - “Variated” shifted Maxwellian velocity distribution
    \[ f_{1,\text{sh},\Delta v}(v) = \frac{1}{\sqrt{\pi}} \left[ \frac{v}{v_0 (v_0 + \Delta v)} \right] \left\{ e^{-\frac{[v-(v_0+\Delta v)]^2}{v_0^2}} - e^{-\frac{[v+(v_0+\Delta v)]^2}{v_0^2}} \right\} \]
  - Simple Maxwellian velocity distribution
    \[ f_{1,\text{Gau}}(v) = \frac{4}{\sqrt{\pi}} \left( \frac{v^2}{v_0^3} \right) e^{-v^2/v_0^2} \]
  - “Modified” simple Maxwellian velocity distribution
    \[ f_{1,\text{Gau},k}(v) = \frac{v^2}{N_{f,k}} \left( e^{-v^2/kv_0^2} - e^{-v_{\text{max}}^2/kv_0^2} \right)^k \quad \text{for } v \leq v_{\text{max}} \]
Bayesian reconstruction of the 1-D WIMP velocity distribution

- Reconstructed $f_{1, \text{Bayesian}}(v)$
  
  $(^{76}\text{Ge}, \text{2.5 - 50 keV}, b_1 = 2.5 \text{ keV}, 500 \text{ events}, m_\chi = 20 \text{ GeV})$ $f_{1, \text{sh}, v_0}(v)$, flat-dist.)

[CLS, IJMPD 24, 1550090 (2015); Y. Bai, W. Sun and CLS (2018)]
Bayesian reconstruction of the 1-D WIMP velocity distribution

- Reconstructed $f_{1, \text{Bayesian}}(v)$
  
  $^{76}\text{Ge}$, 5 - 50 keV, $b_1 = 2.5$ keV, 500 events, $m_\chi = 20$ GeV $f_{1, \text{sh, } v_0}(v)$, flat-dist.)

[CLS, IJMPD 24, 1550090 (2015); Y. Bai, W. Sun and CLS (2018)]
Bayesian reconstruction of the 1-D WIMP velocity distribution

- Reconstructed $f_{1,Bayesian}(v)$
  - $(^{28}\text{Si}, 2.5 - 50$ keV, $b_1 = 5$ keV, 500 events, $m_\chi = 20$ GeV $f_{1,sh,v0}(v)$, flat-dist.)

![Graph showing Bayesian reconstruction of the 1-D WIMP velocity distribution]

[CLS, IJMPD 24, 1550090 (2015); Y. Bai, W. Sun and CLS (2018)]
Bayesian reconstruction of the 1-D WIMP velocity distribution

- Reconstructed \( f_{1,\text{Bayesian}}(v) \)
  
  \[
  (^{28}\text{Si}, \text{5 - 50 keV}, b_1 = 5 \text{ keV}, 500 \text{ events}, m_\chi = 20 \text{ GeV} \ f_{1,\text{sh},v_0}(v), \text{flat-dist.})
  \]

[CLS, IJMPD 24, 1550090 (2015); Y. Bai, W. Sun and CLS (2018)]
Determination of the WIMP mass
Determination of the WIMP mass

**Estimating the moments of the WIMP velocity distribution**

\[
\langle v^n \rangle = \alpha^n \left[ \frac{2Q_{\min}^{1/2} r^*(Q_{\min})}{F^2(Q_{\min})} + l_0(Q_{\min}, Q_{\max}) \right]^{-1} \left[ \frac{2Q_{\min}^{(n+1)/2} r^*(Q_{\min})}{F^2(Q_{\min})} + (n + 1)l_n(Q_{\min}, Q_{\max}) \right]
\]

\[
r^*(Q_{\min}) \equiv r(Q_{\min}) \left[ K_1(Q_{\min}) Q_{\min} + 1 \right]
\]

[Ref: M. Drees and CLS, JCAP 0806, 012 (2008); Y. Bai, W. Sun and CLS (2018)]

**Determining the WIMP mass**

\[
m_X|_{\langle v^n \rangle} = \frac{\sqrt{m_X m_Y} - m_X(\mathcal{R}_{n,\chi}/\mathcal{R}_{n,\gamma})}{\mathcal{R}_{n,\chi}/\mathcal{R}_{n,\gamma} - \sqrt{m_X/m_Y}}
\]

\[
\mathcal{R}_{n,\chi} \equiv \left[ \frac{2Q_{\min,\chi}^{(n+1)/2} r^*_\chi(Q_{\min,\chi})/F^2_X(Q_{\min,\chi}) + (n + 1)l_n,\chi}{2Q_{\min,\chi}^{1/2} r^*_\chi(Q_{\min,\chi})/F^2_X(Q_{\min,\chi}) + l_0,\chi} \right]^{1/n} \quad (n \neq 0)
\]


**Assuming a dominant SI scalar WIMP-nucleus interaction**

\[
m_X|_{\sigma} = \left( \frac{m_X/m_\gamma}{m_Y} \right)^{5/2} \frac{m_Y - m_X(\mathcal{R}_{\sigma,\chi}/\mathcal{R}_{\sigma,\gamma})}{\mathcal{R}_{\sigma,\chi}/\mathcal{R}_{\sigma,\gamma} - (m_X/m_\gamma)^{5/2}}
\]

\[
\mathcal{R}_{\sigma,\chi} \equiv \frac{1}{\mathcal{E}_\chi} \left[ \frac{2Q_{\min,\chi}^{1/2} r^*_\chi(Q_{\min,\chi})}{F^2_X(Q_{\min,\chi})} + l_0,\chi \right]
\]

[Ref: M. Drees and CLS, JCAP 0806, 012 (2008); Y. Bai, W. Sun and CLS (2018)]
Determination of the WIMP mass

- Reconstructed $m_{\chi, \text{rec}}$
  
  $^{28}\text{Si} + ^{76}\text{Ge}$, $Q_{\text{max}} < 100$ keV, $b_1 = 10$ keV, $2 \times 50$ events

![Graph showing the relationship between $m_{\chi, \text{rec}}$ and $m_{\chi, \text{in}}$.]

[Y. Bai, W. Sun and CLS (2018)]

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Direct Detection and Identification of WIMP Dark Matter

- Model-independent WIMP identification
- Determination of the WIMP mass

Determination of the WIMP mass

- Reconstructed $m_{\chi, \text{rec}}$
  
  $^{28}\text{Si} + ^{76}\text{Ge}, \ Q_{\text{max}} < 100 \text{ keV}, \ b_1 = 20 \text{ keV}, \ 2 \times 50 \text{ events}$

[Image of a graph showing the relationship between $m_{\chi, \text{rec}}$ and $m_{\chi, \text{in}}$ for different values of $Q_{\text{min}}$.]

[Y. Bai, W. Sun and CLS (2018)]
Estimation of the SI scalar WIMP-nucleon coupling
Estimation of the SI scalar WIMP-nucleon coupling

- Spin-independent (SI) scalar WIMP-nucleus cross section

\[ \sigma_{0}^{\text{SI}} = \left( \frac{4}{\pi} \right) m_{r,N}^2 [Z f_p + (A - Z) f_n]^2 \approx \left( \frac{4}{\pi} \right) m_{r,N}^2 A^2 |f_p|^2 = A^2 \left( \frac{m_{r,N}}{m_{r,p}} \right)^2 \sigma_{\chi p}^{\text{SI}} \]

\[ \sigma_{\chi p}^{\text{SI}} = \left( \frac{4}{\pi} \right) m_{r,p}^2 |f_p|^2 \]

- Rewrite the integral over \( f_1(v)/v \)

\[ \left( \frac{dR}{dQ} \right)_{\text{expt}, Q=Q_{\text{min}}} = \frac{\varepsilon \rho_0 A^2}{2m_\chi m_{r,p}^2} \left[ \left( \frac{4}{\pi} \right) m_{r,p}^2 |f_p|^2 \right] F^2(Q_{\text{min}}) \left\{ m_{r,N} \sqrt{\frac{2}{m_N}} \left[ \frac{2Q_{\text{min}}^{1/2} r^*_{Q_{\text{min}}}}{F^2(Q_{\text{min}})} + l_0 \right] \right\}^{-1} \left[ \frac{2r(Q_{\text{min}})}{F^2(Q_{\text{min}})} \right] \]

- Estimating the SI scalar WIMP-nucleon coupling

\[ |f_p|^2 = \frac{1}{\rho_0} \left[ \frac{\pi}{4\sqrt{2}} \left( \frac{1}{\varepsilon Z A_Z^2 \sqrt{m_Z}} \right) \right] \left[ \frac{2Q_{\text{min}}^{1/2} r^*_{Q_{\text{min}},Z}}{F_{Z}^2(Q_{\text{min}},Z)} + l_{0,Z} \right] (m_\chi + m_Z) \]

Estimation of the SI scalar WIMP-nucleon coupling

- Reconstructed $|f_p|^2_{\text{rec}}$
  
  ($^{76}\text{Ge}$, $Q_{\text{max}} < 100$ keV, 50 events, $\sigma_{\chi p}^{\text{SI}} = 10^{-9}$ pb)

![Graph showing the estimation of the SI scalar WIMP-nucleon coupling](image)

[Y. Bai, W. Sun and CLS (2018)]
Direct Detection and Identification of WIMP Dark Matter

- Model-independent WIMP identification
- Determinations of ratios of WIMP-nucleon cross sections

Determination of the ratio of SD WIMP-nucleon couplings

- **Spin-dependent (SD) axial-vector** WIMP-nucleus cross section

\[ \sigma_{0}^{SD} = \left( \frac{32}{\pi} \right) G_{F}^{2} m_{r,N}^{2} \left( \frac{J + 1}{J} \right) [\langle S_{p} \rangle a_{p} + \langle S_{n} \rangle a_{n}]^{2} \]

\[ \sigma_{\chi p/n}^{SD} = \left( \frac{32}{\pi} \right) G_{F}^{2} m_{r,p/n}^{2} \cdot \left( \frac{3}{4} \right) a_{p/n}^{2} \]

\( J \): total nuclear spin

\( \langle S_{(p,n)} \rangle \): expectation values of the proton/neutron group spin

\( a_{(p,n)} \): effective SD axial-vector WIMP-proton/neutron couplings

- Determining the **ratio of two SD axial-vector WIMP-nucleon couplings**

\[ \left( \frac{a_{n}}{a_{p}} \right)_{\pm,n}^{SD} = - \frac{\langle S_{p} \rangle_{X} \pm \langle S_{p} \rangle_{Y} (\mathcal{R}_{J,n,X}/\mathcal{R}_{J,n,Y})}{\langle S_{n} \rangle_{X} \pm \langle S_{n} \rangle_{Y} (\mathcal{R}_{J,n,X}/\mathcal{R}_{J,n,Y})} \]

\[ \mathcal{R}_{J,n,X} \equiv \left[ \left( \frac{J_{X}}{J_{X} + 1} \right) \frac{\mathcal{R}_{\sigma,X}}{\mathcal{R}_{n,X}} \right]^{1/2} \quad (n \neq 0) \]

[M. Drees and CLS, arXiv:0903.3300]
Determinations of the ratio of SD WIMP-nucleon couplings

- Reconstructed \((a_n/a_p)^{SD}_{\text{rec},1}\)

\((^{19}\text{F} + ^{127}\text{I}, Q_{\text{max}} < 100 \text{ keV}, 2 \times 50 \text{ events, } \sigma_{\chi p}^{\text{SI}} = 1 \text{ zb, } m_{\chi} = 20 \text{ GeV})\)

[Y. Bai, W. Sun and CLS (2018)]
Reconstructed \((a_n/a_p)_{\text{SD}}^{\text{rec},1}\) 
\((^{19}\text{F} + ^{127}\text{I}, Q_{\text{max}} < 100 \text{ keV}, 2 \times 50 \text{ events}, \sigma_{\chi p}^{\text{SI}} = 1 \text{ zb}, a_p = 0.1, a_n/a_p = 0.7)\)

[Y. Bai, W. Sun and CLS (2018)]
Determinations of ratios of WIMP-nucleon cross sections

- Differential rate for combined SI and SD cross sections
  \[
  \left( \frac{dR}{dQ} \right)_{\text{expt}, Q=Q_{\text{min}}} = \mathcal{E} \left( \frac{\rho_0 \sigma_{0}^{\text{SI}}}{2m_N m^2} \right) \left[ F_{\text{SI}}^2(Q) + \left( \frac{\sigma_{\chi p}^{\text{SD}}}{\sigma_{\chi p}^{\text{SI}}} \right) C_p F_{\text{SD}}^2(Q) \right] \int_{v_{\text{min}}}^{v_{\text{max}}} \left[ \frac{f_1(v)}{v} \right] dv
  \]

  \[ C_p \equiv \frac{4}{3} \left( \frac{J + 1}{J} \right) \left[ \frac{\langle S_p \rangle + (a_n/a_p) \langle S_n \rangle}{A} \right]^2 \]

- Determining the ratio of two WIMP-proton cross sections
  \[
  \frac{\sigma_{\chi p}^{\text{SD}}}{\sigma_{\chi p}^{\text{SI}}} = \frac{F_{\text{SI}, Y}(Q_{\text{min}}, Y)(R_{m,X}/R_{m,Y}) - F_{\text{SI}, X}(Q_{\text{min}}, X)}{C_p,X F_{\text{SD}, X}(Q_{\text{min}}, X) - C_p,Y F_{\text{SD}, Y}(Q_{\text{min}}, Y)(R_{m,X}/R_{m,Y})}
  \]

  \[ R_{m,X} \equiv \frac{r_X^*(Q_{\text{min}}, X)}{\mathcal{E}_X A_X^2} \]

- Determining the ratio of two SD axial-vector WIMP-nucleon couplings
  \[
  \left( \frac{a_n}{a_p} \right)^{\text{SI+SD}}_{\pm} = - \left( c_p,X s_{n/p,X} - c_p,Y s_{n/p,Y} \right) \pm \sqrt{c_p,X c_p,Y} \left| s_{n/p,X} - s_{n/p,Y} \right|
  \]

  \[ c_p,X \equiv \frac{4}{3} \left( \frac{J_X + 1}{J_X} \right) \left[ \frac{\langle S_p \rangle X}{A_X} \right]^2 \left[ F_{\text{SI}, Z}(Q_{\text{min}}, Z) \left( \frac{R_{m,Y}}{R_{m,Z}} \right) - F_{\text{SI}, Y}(Q_{\text{min}}, Y) \right] F_{\text{SD}, X}(Q_{\text{min}}, X) \]


\[ C.-L. Shan (XAO-CAS) \]

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Determinations of ratios of WIMP-nucleon cross sections

- Reconstructed \( \left( \frac{a_n}{a_p} \right)_{\text{rec}}^{\text{SI}+\text{SD}} \) vs. \( \left( \frac{a_n}{a_p} \right)_{\text{rec},1}^{\text{SD}} \)

\[ ^{19}\text{F} + ^{127}\text{I} (+ ^{28}\text{Si}), 5 - 100 \text{ keV}, 2(3) \times 50 \text{ events}, \sigma_{\chi p}^{\text{SI}} = 1 \text{ zb}, m_\chi = 20 \text{ GeV} \]

\[ [\text{Y. Bai, W. Sun and CLS (2018)}] \]
Determinations of ratios of WIMP-nucleon cross sections

- Reconstructed $(a_n/a_p)^{SI+SD}_{rec}$ vs. $(a_n/a_p)^{SD}_{rec,1}$
  
  $(^{19}F + ^{127}I (+ ^{28}Si))$, 5 – 100 keV, 2(3) × 50 events, $\sigma_{Xp}^{SI} = 1$ zb, $a_n/a_p = 0.7$

[Y. Bai, W. Sun and CLS (2018)]
Determinations of ratios of WIMP-nucleon cross sections

- Reconstructed \((a_n/a_p)^{SI+SD}\)_{rec}

\((^{19}F + ^{127}I + ^{28}Si, \ Q_{\text{max}} < 100 \text{ keV}, \ 3 \times 50 \text{ events}, \ \sigma_{^p}^{SI} = 1 \text{ zb}, \ a_n/a_p = 0.7)\)

[Y. Bai, W. Sun and CLS (2018)]
Determinations of ratios of WIMP-nucleon cross sections

- Reconstructed \( \left( \frac{\sigma^{SD}_{\chi p}}{\sigma^{SI}_{\chi p}} \right)_{\text{rec}} \)
  \( ^{19}\text{F} + ^{127}\text{I} + ^{28}\text{Si}, Q_{\text{max}} < 100 \text{ keV}, 3 \times 50 \text{ events}, \sigma^{SI}_{\chi p} = 1 \text{ zb}, a_p = 0.1 \)
Determinations of ratios of WIMP-nucleon cross sections

- Reconstructed \( \left( \frac{\sigma_{\chi p}^{SD}}{\sigma_{\chi p}^{SI}} \right)_{\text{rec}} \)
- \( ^{23}\text{Na} + ^{76}\text{Ge} \), \( Q_{\text{max}} < 100 \text{ keV} \), 2 × 50 events, \( \sigma_{\chi p}^{SI} = 1 \text{ zb} \), \( a_p = 0.1 \)

\[ Y. \text{ Bai, W. Sun and CLS (2018)} \]
Determinations of ratios of WIMP-nucleon cross sections

- Reconstructed \( \left(\frac{\sigma_{\chi n}^{SD}}{\sigma_{\chi p}^{SI}}\right)_{\text{rec}} \)

\[ (^{19}\text{F} + ^{127}\text{I} + ^{28}\text{Si}, \ Q_{\text{max}} < 100 \text{ keV}, \ 3 \times 50 \text{ events, } \sigma_{\chi p}^{SI} = 1 \text{ zb, } a_p = 0.1, a_n/a_p = 0.7) \]

[Y. Bai, W. Sun and CLS (2018)]
Determinations of ratios of WIMP-nucleon cross sections

- Reconstructed \( \left( \frac{\sigma_{SD}^{\chi_n}}{\sigma_{SI}^{\chi_p}} \right)_{rec} \)
  \( ^{131}\text{Xe} + ^{76}\text{Ge}, Q_{\text{max}} < 100\text{ keV}, 2 \times 50\text{ events}, \sigma_{SI}^{\chi_p} = 1\text{ zb}, a_p = 0.1, a_n/a_p = 0.7 \)
Summary
Summary

- Astronomical observations and measurements indicate the existence of Dark Matter
  - Rotation curves of spiral galaxies
  - Anisotropy of the CMB radiation

- Models in particle physics offer candidates for Dark Matter
  - the lightest neutralino in SUSY models
  - the lightest Kaluza-Klein (KK) particle in UED models
  - the lightest T-odd particle in Little Higgs models

- We are searching for Dark Matter by
  - producing new particle(s) at colliders
  - indirect detection of secondary products of WIMP annihilation/decay
  - direct detection through elastic WIMP-nucleus scattering
Summary

Once two or more experiments with different target nuclei observe positive WIMP signals, we could reconstruct

- WIMP mass $m_\chi$
- 1-D velocity distribution $f_1(v)$
- SI WIMP-proton coupling $|f_p|^2$ (with an estimated $\rho_0$)
- ratio between the SD WIMP-nucleon couplings $a_n/a_p$
- ratios between the SD and SI WIMP-nucleon cross sections $\sigma^{SD}_{\chi(p,n)}/\sigma^{SI}_{\chi p}$

With a predicted $f_{1,th}(v)$, one can fit $f_1(v)$ by using Bayesian analysis.

For these analyses, the local density, the velocity distribution, and the mass or couplings on nucleons of halo WIMPs are not required priorly.

For a WIMP mass of $O(100 \text{ GeV})$, with only $O(50)$ events from one experiment and less than $\sim 20\%$ unrejected backgrounds, these quantities could be estimated with statistical uncertainties of $10\% - 40\%$. 
Thank you very much for your attention!