Introduction to
Direct Dark Matter Detection Experiments

Chung-Lin Shan
Xinjiang Astronomical Observatory
Chinese Academy of Sciences

School of Physics and Engineering, Sun Yat-Sen University
September 17, 2015
Evidence and candidates for Dark Matter
   Evidence for Dark Matter
   Candidates for Dark Matter

Direct Dark Matter detection
   Elastic WIMP-nucleus scattering
   WIMP-nucleus cross sections
   Detection techniques
   Background discrimination
Evidence for Dark Matter
Evidence for Dark Matter

- Rotation curves of spiral galaxies (1970s)

Evidence for Dark Matter

- Collision of two clusters of galaxies (Bullet Cluster, 1E 0657-56)

[http://chandra.harvard.edu/photo/2006/1e0657/]
Evidence for Dark Matter

- Collision of two clusters of galaxies (Bullet Cluster, 1E 0657-56)

[http://chandra.harvard.edu/photo/2006/1e0657/]

Evidence for Dark Matter

- Collision of two clusters of galaxies (Bullet Cluster, 1E 0657-56)

[http://chandra.harvard.edu/photo/2006/1e0657/]
Evidence for Dark Matter

- Anisotropy of the cosmic microwave background radiation (CMBR)

[NASA/WMAP Science Team (full sky map in 1965)]
Evidence for Dark Matter

- Anisotropy of the cosmic microwave background radiation (CMBR)

Evidence for Dark Matter

- Anisotropy of the cosmic microwave background radiation (CMBR)

[NASA/WMAP Science Team, WMAP 5-year result (2008)]
Evidence for Dark Matter

- Anisotropy of the cosmic microwave background radiation (CMBR)

Evidence for Dark Matter

- A large fraction of the mass/energy in our Universe is dark!

  ▲ Dark Energy: $68.5(^{+1.7}_{-1.6})\%$
  
  ▲ Dark Matter: $26.5(\pm1.1)\%$
  
  ▲ Baryonic matter: $4.99(\pm0.22)\%$
  
  ▲ Luminous matter: $0.357(\pm0.006)\%$
  
  ▲ Stars: $0.2\% \sim 0.3\%$
  
  ▲ Neutrinos: $< 0.55\%$
  
  ▲ CMB photons: $0.00546(\pm0.00019)\%$

[‡: Review of Particle Physics 2014, 23. Big–Bang Nucleosynthesis]
[§: Review of Particle Physics 2014, 22. Big–Bang Cosmology]
Candidates for Dark Matter
Candidates for Dark Matter

Particles of the Standard Model

## Candidates for Dark Matter

### Particles of the Standard Model

<table>
<thead>
<tr>
<th>SM particles</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Symbol</td>
</tr>
<tr>
<td>up-quarks</td>
<td>u, c, t</td>
</tr>
<tr>
<td>down-quarks</td>
<td>d, s, b</td>
</tr>
<tr>
<td>leptons</td>
<td>e, μ, τ</td>
</tr>
<tr>
<td>neutrinos</td>
<td>νe, νμ, ντ</td>
</tr>
<tr>
<td>gluons</td>
<td>g</td>
</tr>
<tr>
<td>photon</td>
<td>γ</td>
</tr>
<tr>
<td>Z boson</td>
<td>Z⁰</td>
</tr>
<tr>
<td>Higgs boson</td>
<td>H</td>
</tr>
<tr>
<td>W bosons</td>
<td>W⁺⁻</td>
</tr>
</tbody>
</table>
Candidates for Dark Matter

Particles of typical **supersymmetric models**

<table>
<thead>
<tr>
<th>Normal particles</th>
<th>SUSY partners</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>up-quarks</strong></td>
<td></td>
</tr>
<tr>
<td>u, c, t</td>
<td>up-squarks</td>
</tr>
<tr>
<td>d, s, b</td>
<td>down-squarks</td>
</tr>
<tr>
<td><strong>down-quarks</strong></td>
<td></td>
</tr>
<tr>
<td>e, μ, τ</td>
<td>sleptons</td>
</tr>
<tr>
<td>ν_e, ν_μ, ν_τ</td>
<td>sneutrinos</td>
</tr>
<tr>
<td><strong>leptons</strong></td>
<td></td>
</tr>
<tr>
<td><strong>neutrinos</strong></td>
<td></td>
</tr>
<tr>
<td><strong>gluons</strong></td>
<td></td>
</tr>
<tr>
<td><strong>photons</strong></td>
<td></td>
</tr>
<tr>
<td>Z boson</td>
<td></td>
</tr>
<tr>
<td>light scalar Higgs</td>
<td></td>
</tr>
<tr>
<td>heavy scalar Higgs</td>
<td></td>
</tr>
<tr>
<td>pseudoscalar Higgs</td>
<td></td>
</tr>
<tr>
<td>charged Higgs</td>
<td></td>
</tr>
<tr>
<td>W bosons</td>
<td></td>
</tr>
<tr>
<td>graviton</td>
<td></td>
</tr>
<tr>
<td>gravitino</td>
<td></td>
</tr>
<tr>
<td>axion</td>
<td></td>
</tr>
<tr>
<td>axino</td>
<td></td>
</tr>
</tbody>
</table>

*Weakly Interacting Massive Particles (WIMPs)*

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Name</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>up-squarks</td>
<td>u_L, u_R, c_L, c_R, t_L, t_R</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>d_L, d_R, s_L, s_R, b_L, b_R</td>
<td></td>
<td></td>
</tr>
<tr>
<td>sneutrinos</td>
<td>v_e, v_μ, v_τ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>neutralinos</td>
<td>χ_0^0, χ^-_1, χ^+_1, χ_2, χ_3, χ_4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>neutralinos</td>
<td>χ^-_1, χ^+_2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>gravitino</td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>gravitino</td>
<td>G</td>
<td></td>
<td></td>
</tr>
<tr>
<td>axino</td>
<td>a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>axino</td>
<td>a</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Direct Dark Matter detection
Dark Matter searches

DM should have small, but non-zero interactions with ordinary matter. 

⇒ Three different ways to detect DM particles

- **Colliders**
  - $(p, e)$
  - $p^-, e^+$

- **Indirect detection**
  - $e, \nu_\mu, \gamma$

- **Direct detection**
  - $e^+, \bar{p}, \bar{D}$
Direct Dark Matter detection: elastic WIMP-nucleus scattering

- WIMPs could scatter elastically off target nuclei and produce nuclear recoils which deposit energy in the detector.

  ▲ The event rate depends on
  ▶ the WIMP density near the Earth $\rho_0$
  ▶ the WIMP-nucleus cross sections $\sigma_0^{SI}$ and $\sigma_0^{SD}$
  ▶ the WIMP mass $m_\chi$
  ▶ the velocity distribution of incident WIMPs $f_1(v)$

  ▲ The WIMP-nucleus cross section is about $10^{-2} \sim 10^{-6}$ pb,
   ▶ the optimistic expected event rate is $\sim 10^{-4}$ events/kg-day
   ▶ but could be $< 1$ event/ton-yr

  ▲ An exponential-like recoil energy spectrum
   ▶ Most events would be with energies less than 50 keV

  ▲ Typical background events due to cosmic rays and ambient radioactivity: signals $\approx \mathcal{O}(10^6) : 1 = \text{bg discrimination} \Rightarrow \sim 1 : 1.x$
WIMP-nucleus cross sections

- Target material dependence
  - Spin-independent (SI) coupling
    - a scalar (and/or vector) interaction
    - the cross section for scalar interaction is approximately proportional to the square of the mass of the target nucleus.
    - Heavier nuclei, e.g. Ge and Xe, are usually used.
  - Spin-dependent (SD) coupling
    - an axial-vector (spin-spin) interaction
    - $^{19}$F, $^{127}$I, and $^{129/131}$Xe are usually used.
  - For nuclei with $A \geq 30$, the SI interaction almost always dominates over the SD interaction.
  - The scattering event rate and the detector sensitivity depend on the mass of target nuclei directly.
WIMP-nucleus cross sections

- Exclusion limits on the (predicted) SI WIMP-nucleon cross section

[http://dmtools.berkeley.edu/limitplots/]
Detection techniques

- Induced signals
  - Ionization (charges)
  - Scintillation (light)
  - Heat (phonons)
  - Quenching factor (nuclear recoil relative efficiency)
    measured (electron equivalent) recoil energy $\text{keV}_{ee}$
    true recoil energy $\text{keV}_r$
  - Raw/total mass/exposure, fiducial mass/exposure
  - Combinations of two signals
    a powerful event-by-event rejection method for the background discrimination down to 5 to 10 keV recoil energy
Detection techniques

- Semiconductor/scintillator detectors

  ▲ **ANAIS**
  NaI(Tl), Laboratorio Subterráneo de Canfranc (LSC), Spain.

  ▲ **CDEX-TEXONO**
  Ge, China Jin-Ping Laboratory (CJPL), China.

  ▲ **CDMS**
  Ge and Si, Soudan Underground Laboratory, Minnesota, USA.

  ▲ **CoGeNT**
  Ge, Soudan Underground Laboratory, Minnesota, USA.

  ▲ **CRESST**
  Al₂O₃/CaWO₄, Laboratori Nazionali del Gran Sasso (LNGS), Italy.

  ▲ **DAMA**
  NaI(Tl), LNGS, Italy.

  ▲ **DM-Ice**
  NaI(Tl), South Pole.

  ▲ **EDELWEISS (EDW)**
  Ge, Laboratoire Souterrain de Modane (LSM), France.

  ▲ **HDMS**
  Ge, LNGS, Italy.

  ▲ **KIMS**
  CsI(Tl), Yangyang Laboratory (Y2L), South Korea.

  ▲ **NaIAD**
  NaI(Tl), Boulby Underground Laboratory, UK.
Detection techniques

- Liquid noble gas detectors
  - ArDM
    Dual-phase (gas-liquid) Ar, CERN, Switzerland.
  - DarkSide
    Dual-phase Ar, LNGS, Italy.
  - DARWIN
    Dual-phase Ar and Xe, ULISSE, France or LNGS, Italy.
  - DEAP/CLEAN
    Single-phase Ar and Ne, SNO Underground Laboratory, Canada.
  - LUX
    Dual-phase Xe, USA.
  - MAX
    Dual-phase Ar and Xe, USA.
  - PandaX
    Xe, CJPL, China.
  - WARP
    Dual-phase Ar, LNGS, Italy.
  - XENON
    Dual-phase Xe, LNGS, Italy.
  - XMASS
    Single-phase Xe, SuperKamiokande, Japan.
  - ZEPLIN
    Single-/dual-phase Xe, Boulby Underground Laboratory, UK.
Detection techniques

1. Superheated droplet/gas detectors

- **COUPP**
  Bubble chamber, CF$_3$I, C$_3$F$_8$, and C$_4$F$_{10}$, USA.

- **D3**
  Hawaii, USA

- **DAMIC**
  Si, SNO Underground Laboratory, Canada.

- **DMTPC**
  CF$_4$, USA.

- **DRIFT**
  Xe-CS$_2$, Boulby Underground Laboratory, UK.

- **MIMAC**
  CF$_4$, Laboratoire de Physique Subatomique et de Cosmologie (LPSC), France.

- **NEWAGE**
  CF$_4$, Kamioka mine, Japan.

- **NEWS**
  Laboratoire Souterrain de Modane (LSM), France.

- **Nuclear Emulsion**
  Nagoya, Japan.

- **PICASSO**
  Bubble chamber, C$_4$F$_{10}$, SNO Underground Laboratory, Canada.

- **SIMPLE**
  C$_2$ClF$_5$, CF$_3$I, Laboratoire Souterrain à Bas Bruit (LSBB), France.
Background discrimination

- Background and background discrimination
  - Cosmic muons
  - External natural radioactivity
  - Internal natural radioactivity
  - Fast neutrons
  - Neutron-induced nuclear recoils
  - Multiple-scatter events
  - Electron recoils
  - Surface events
  - Incomplete charge collection
Background discrimination

- Background and background discrimination

  ▲ Cosmic muons
    ▶ Induce fast neutrons
    ▶ $\mathcal{O}(10^{10})$ cosmic muons/m$^2$ Earth’s surface/yr
    ▶ Go deep underground (reduced by a factor of $10^5$ to $10^8$)

Background discrimination

- Background and background discrimination
  - External natural radioactivity
    - Radioactive isotopes in the rock/walls
    - Passive shielding:
      - lead (high-Z materials) for MeV $\gamma$-ray,
      - (low-Z materials) for $\alpha$, $\beta$, and low energy $\gamma$-rays
  - Internal natural radioactivity
    - Radioactive isotopes contamination in the outer shielding, equipment around the detector, and detector material
    - Radiopure materials
  - Fast neutrons
    - Induced by cosmic-ray in the inner lead shielding
    - Water tank or polyethylene (PE)
      (materials with high density of hydrogen)
Background discrimination

- Background and background discrimination
  - Multiple-scatter events
    - Mean free path of WIMP-induced events $\sim O$ (light year)
    - Array of detectors
    - CDMS ZIP-detector cryostat and tower

[CDMS homepage; P. Cushman, J. Phys. Conf. Ser. 39, 63 (2006)]
Background discrimination

- Background and background discrimination

  ▲ Electron recoils

    ▶ Ionization yield
    ▶ Ionization (S2)/primary scintillation (S1)
    ▶ CDMS-II calibration

[CDMS Collab., Z. Ahmed et al., Science 327, 1619 (2010)]
Background discrimination

- Background and background discrimination

  ▶ Surface events
    - Rising time of phonon pulses
    - Self-shielding
    - CDMS-II calibration

[CDMS Collab., Z. Ahmed et al., Science 327, 1619 (2010)]
Background discrimination

- Background and background discrimination
  - Surface events
    - Rising time of phonon pulses
    - Self-shielding
    - XENON10 result

[XENON10 Collab., J. Angle et al., PRL 100, 021303 (2008)]
Background discrimination

- Background and background discrimination

  ▲ Neutron-induced nuclear recoils
    ▶ Mimic WIMP-induced nuclear recoils
    ▶ Neutron veto
    ▶ Boron-loaded liquid scintillator neutron veto [arXiv:1509.02782]
Background discrimination

- Background and background discrimination
  - Neutron-induced nuclear recoils
    - Mimic WIMP-induced nuclear recoils
    - Scintillating reflector
    - CRESST-II calibration

[CRESST Collab., R. F. Lang et al., Astropart. Phys. 33, 60 (2010)]
Model-Independent Reconstruction of Dark Matter Properties with Direct Detection Experiments

Chung-Lin Shan

Xinjiang Astronomical Observatory
Chinese Academy of Sciences

School of Physics and Engineering, Sun Yat-Sen University
September 17, 2015
Motivation

Reconstruction of the 1-D WIMP velocity distribution
   With measured recoil energies
   With a non-negligible threshold energy
   Bayesian reconstruction of the WIMP velocity distribution function

Determination of the WIMP mass

Determinations of the WIMP-nucleon couplings
   Estimation of the SI scalar WIMP-nucleon coupling
   Determinations of ratios of WIMP-nucleon cross sections

Summary
Motivation
Motivation

- Differential event rate for elastic WIMP-nucleus scattering

\[
\frac{dR}{dQ} = AF^2(Q) \int_{v_{\text{min}}(Q)}^{v_{\text{max}}} \left( \frac{f_1(v)}{v} \right) dv
\]

Here

\[v_{\text{min}}(Q) = \alpha \sqrt{Q}\]

is the minimal incoming velocity of incident WIMPs that can deposit the recoil energy \(Q\) in the detector,

\[A \equiv \frac{\rho_0 \sigma_0}{2m_\chi m_{r,N}^2}, \quad \alpha \equiv \sqrt{\frac{m_N}{2m_{r,N}^2}}, \quad m_{r,N} = \frac{m_\chi m_N}{m_\chi + m_N}\]

\(\rho_0\): WIMP density near the Earth

\(\sigma_0\): total cross section ignoring the form factor suppression

\(F(Q)\): elastic nuclear form factor

\(f_1(v)\): one-dimensional velocity distribution of halo WIMPs
Motivation

- Differential event rate for elastic WIMP-nucleus scattering

\[
\frac{dR}{dQ} = A F^2(Q) \int_{v_{\text{min}}(Q)}^{v_{\text{max}}} \left[ \frac{f_1(v)}{v} \right] dv
\]

Here

\[v_{\text{min}}(Q) = \alpha \sqrt{Q}\]

is the minimal incoming velocity of incident WIMPs that can deposit the recoil energy \(Q\) in the detector,

\[A \equiv \frac{\rho_0 \sigma_0}{2 m_\chi m_{r,N}^2} \quad \alpha \equiv \sqrt{\frac{m_N}{2 m_{r,N}^2}} \quad m_{r,N} = \frac{m_\chi m_N}{m_\chi + m_N}\]

- \(\rho_0\): WIMP density near the Earth
- \(\sigma_0\): total cross section ignoring the form factor suppression
- \(F(Q)\): elastic nuclear form factor
- \(f_1(v)\): one-dimensional velocity distribution of halo WIMPs
Reconstruction of the 1-D WIMP velocity distribution
With measured recoil energies
Reconstruction of the 1-D WIMP velocity distribution

- Normalized one-dimensional WIMP velocity distribution function

\[ f_1(v) = \mathcal{N} \left\{ -2Q \cdot \frac{d}{dQ} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] \right\}_{Q=v^2/\alpha^2} \]

\[ \mathcal{N} = \frac{2}{\alpha} \left\{ \int_0^\infty \frac{1}{\sqrt{Q}} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] dQ \right\}^{-1} \]

- Moments of the velocity distribution function

\[ \langle v^n \rangle = \mathcal{N}(Q_{\text{thre}}) \left( \frac{\alpha^{n+1}}{2} \right) \left\{ \frac{2Q_{\text{thre}}^{(n+1)/2}}{F^2(Q_{\text{thre}})} \left( \frac{dR}{dQ} \right) \right\}_{Q=Q_{\text{thre}}}^{Q_{\text{thre}}} + (n+1) I_n(Q_{\text{thre}}) \]

\[ \mathcal{N}(Q_{\text{thre}}) = \frac{2}{\alpha} \left\{ \frac{2Q_{\text{thre}}^{1/2}}{F^2(Q_{\text{thre}})} \left( \frac{dR}{dQ} \right) \right\}_{Q=Q_{\text{thre}}}^{Q_{\text{thre}}} + I_0(Q_{\text{thre}}) \right\}^{-1} \]

\[ I_n(Q_{\text{thre}}) = \int_{Q_{\text{thre}}}^\infty Q^{(n-1)/2} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] dQ \]

[M. Drees and CLS, JCAP 0706, 011 (2007)]
Reconstruction of the 1-D WIMP velocity distribution

- **Ansatz:** the measured recoil spectrum in the $n$th $Q$-bin

$$\left( \frac{dR}{dQ} \right)_{\text{expt}, Q \approx Q_n} \equiv r_n e^{k_n(Q - Q_{s,n})}$$

$$r_n \equiv \frac{N_n}{b_n}$$

- **Logarithmic slope and shifted point** in the $n$th $Q$-bin

$$Q - Q_n |_n \equiv \frac{1}{N_n} \sum_{i=1}^{N_n} (Q_{n,i} - Q_n) = \left( \frac{b_n}{2} \right) \coth \left( \frac{k_n b_n}{2} \right) - \frac{1}{k_n}$$

$$Q_{s,n} = Q_n + \frac{1}{k_n} \ln \left[ \frac{\sinh(k_n b_n/2)}{k_n b_n/2} \right]$$

- **Reconstructing the one-dimensional WIMP velocity distribution**

$$f_1(v_{s,n}) = \mathcal{N} \left[ \frac{2 Q_{s,n} r_n}{F^2(Q_{s,n})} \right] \left[ \frac{d}{dQ} \ln F^2(Q) \bigg|_{Q=Q_{s,n}} - k_n \right]$$

$$\mathcal{N} = \frac{2}{\alpha} \left[ \sum_a \frac{1}{\sqrt{Q_a} F^2(Q_a)} \right]^{-1}$$

$$v_{s,n} = \alpha \sqrt{Q_{s,n}}$$

[M. Drees and CLS, JCAP 0706, 011 (2007)]
Reconstruction of the 1-D WIMP velocity distribution

- Reconstructed $f_{1,\text{rec}}(v_s,n)$
  ($^{76}\text{Ge}$, 500 events, 5 bins, up to 3 bins per window)

\[ \chi^2/\text{dof} = 0.73 \]

[500 events, 5 bins, up to 3 bins per window]

[M. Drees and CLS, JCAP 0706, 011 (2007)]
With a non-negligible threshold energy
Problem!

- Reconstructed $f_{1,\text{rec}}(v_s, n)$ with a non-negligible threshold energy

($^{76}\text{Ge}$, 2 - 50 keV, 500 events, $m_\chi = 25$ GeV)

[CLS, IJMPD 24, 1550090 (2015)]
Modification of the estimator for the normalization constant $\mathcal{N}$

- Consider the non-zero minimal cut-off velocity

$$\mathcal{N} \approx \frac{2}{\alpha} \left[ \frac{2 Q_{\text{min}}^{1/2}}{F^2(Q_{\text{min}})} \left( \frac{dR}{dQ} \right)_{\text{expt}, Q=Q_{\text{min}}} + I_0(Q_{\text{min}}, Q_{\text{max}}^*) \right]^{-1}$$

where

$$\left( \frac{dR}{dQ} \right)_{\text{expt}, Q=Q_{\text{min}}} = r_1 e^{k_1(Q_{\text{min}} - Q_s, 1)} \equiv r(Q_{\text{min}})$$

$$I_n(Q_{\text{min}}, Q_{\text{max}}^*) = \int_{Q_{\text{min}}}^{Q_{\text{max}}^*} Q^{(n-1)/2} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] dQ$$

$$Q_{\text{max}}^* \equiv \min \left( Q_{\text{max}}, Q_{\text{max}, \text{kin}} = \frac{v_{\text{max}}^2}{\alpha^2} \right)$$

[CLS, IJMPD 24, 1550090 (2015)]
Modification of the estimator for the normalization constant $\mathcal{N}$

- Reconstructed $f_{1,\text{rec}}(v_s,n)$ with the input WIMP mass
  $(^{76}\text{Ge}, 2 - 50 \text{ keV, 500 events, } m_\chi = 25 \text{ GeV})$

[CLS, IJMPD 24, 1550090 (2015)]
Modification of the estimator for the normalization constant $\mathcal{N}$

- Height of the velocity distribution at the non-zero minimal cut-off velocity

$$f_{1,\text{rec}}(v^*_\text{min}) = \mathcal{N} \left[ \frac{2Q_{\text{min}} r(Q_{\text{min}})}{F^2(Q_{\text{min}})} \right] \left[ \frac{d}{dQ} \ln F^2(Q) \right]_{Q=Q_{\text{min}}} - k_1 \equiv \mathcal{N} \tilde{f}_{1,\text{rec}}(v^*_\text{min})$$

- Consider the contribution below the non-zero minimal cut-off velocity

$$\mathcal{N} = \frac{2}{\alpha} \left[ \tilde{f}_{1,\text{rec}}(v^*_\text{min}) Q_{\text{min}}^{1/2} + \frac{2Q_{\text{min}}^{1/2}}{F^2(Q_{\text{min}})} \left( \frac{dR}{dQ} \right)_{\text{expt, } Q=Q_{\text{min}}} + l_0(Q_{\text{min}}, Q^*_\text{max}) \right]^{-1}$$
Modification of the estimator for the normalization constant $N$

- Reconstructed $f_{1,\text{rec}}(v_s,n)$ with the input WIMP mass

$^{76}\text{Ge, 2 - 50 keV, 500 events, } m_\chi = 25 \text{ GeV}$

[CLS, IJMPD 24, 1550090 (2015)]
Modification of the estimator for the normalization constant $N$

- Reconstructed $f_{1,rec}(v_s, n)$ with the input WIMP mass
  
  ($^{76}$Ge, 5 - 50 keV, 500 events, $m_{\chi} = 25$ GeV)

[CLS, IJMPD 24, 1550090 (2015)]
Modification of the estimator for the normalization constant $\mathcal{N}$

- Theoretical bias estimate of

$$\left[ \Delta_{0}^{\nu_{\text{min}}^*} - \int_{0}^{\nu_{\text{min}}^*} f_{1}(v) \, dv \right] / \int_{0}^{\nu_{\max}} f_{1}(v) \, dv$$

[CLS, IJMPD 24, 1550090 (2015)]
Bayesian reconstruction of the WIMP velocity distribution function
Formalism

- Bayesian analysis

\[ p(\Theta | \text{data}) = \frac{p(\text{data} | \Theta)}{p(\text{data})} \cdot p(\Theta) \]

- \( \Theta: \{a_1, a_2, \cdots, a_{N_{\text{Bayesian}}}\} \), a specified (combination of the) value(s) of the fitting parameter(s)

- \( p(\Theta) \): prior probability, our degree of belief about \( \Theta \) being the true value(s) of fitting parameter(s), often given in form of the (multiplication of the) probability distribution(s) of the fitting parameter(s)

- \( p(\text{data}) \): evidence, the total probability of obtaining the particular set of data

- \( p(\text{data} | \Theta) \): the probability of the observed result, once the specified (combination of the) value(s) of the fitting parameter(s) happens, usually be described by the “likelihood” function of \( \Theta \), \( L(\Theta) \)

- \( p(\Theta | \text{data}) \): posterior probability density function for \( \Theta \), the probability of that the specified (combination of the) value(s) of the fitting parameter(s) happens, given the observed result
Formalism

- Probability distribution functions for $p(\Theta)$
  - Without prior knowledge about the fitting parameter
    - Flat-distributed
      \[ p_i(a_i) = 1 \quad \text{for} \quad a_{i,\text{min}} \leq a_i \leq a_{i,\text{max}} \]
  - With prior knowledge about the fitting parameter
    - Around a theoretical predicted/estimated or experimental measured value $\mu_{a,i}$
    - With (statistical) uncertainties $\sigma_{a,i}$
    - Gaussian-distributed
      \[ p_i(a_i; \mu_{a,i}, \sigma_{a,i}) = \frac{1}{\sqrt{2\pi} \sigma_{a,i}} \exp\left(-\frac{(a_i - \mu_{a,i})^2}{2\sigma_{a,i}^2}\right) \]

[CLS, JCAP 1408 009 (2014)]
Formalism

- Likelihood function for $p(\text{data}|\Theta)$
  - Theoretical one-dimensional WIMP velocity distribution function:
    $$f_{1,\text{th}}(v; a_1, a_2, \cdots, a_{N_{\text{Bayesian}}})$$
  - Assuming that the reconstructed data points are Gaussian-distributed around the theoretical predictions

$$\mathcal{L}(f_{1,\text{rec}}(v_s,\mu), \mu = 1, 2, \cdots, W; a_i, i = 1, 2, \cdots, N_{\text{Bayesian}})$$

$$\equiv \prod_{\mu=1}^{W} \text{Gau}(v_s,\mu, f_{1,\text{rec}}(v_s,\mu), \sigma_{f_{1,s,\mu}}; a_1, a_2, \cdots, a_{N_{\text{Bayesian}}})$$

with

$$\text{Gau}(v_s,\mu, f_{1,\text{rec}}(v_s,\mu), \sigma_{f_{1,s,\mu}}; a_1, a_2, \cdots, a_{N_{\text{Bayesian}}}) \equiv \frac{1}{\sqrt{2\pi} \sigma_{f_{1,s,\mu}}} e^{-\left[f_{1,\text{rec}}(v_s,\mu) - f_{1,\text{th}}(v_s,\mu; a_1, a_2, \cdots, a_{N_{\text{Bayesian}}})\right]^2 / 2\sigma_{f_{1,s,\mu}}^2}$$

[CLS, JCAP 1408 009 (2014)]
Numerical results

- Input and fitting one-dimensional WIMP velocity distribution functions

  - "One-parameter" shifted Maxwellian velocity distribution
    \[ f_{1,\text{sh},v_0}(v) = \frac{1}{\sqrt{\pi}} \left( \frac{v}{v_0 v_e} \right) \left[ e^{-(v-v_e)^2/v_0^2} - e^{-(v+v_e)^2/v_0^2} \right] \quad v_e = 1.05 \ v_0 \]

  - Shifted Maxwellian velocity distribution
    \[ f_{1,\text{sh}}(v) = \frac{1}{\sqrt{\pi}} \left( \frac{v}{v_0 v_e} \right) \left[ e^{-(v-v_e)^2/v_0^2} - e^{-(v+v_e)^2/v_0^2} \right] \]

  - "Variated" shifted Maxwellian velocity distribution
    \[ f_{1,\text{sh},\Delta v}(v) = \frac{1}{\sqrt{\pi}} \left[ \frac{v}{v_0 (v_0 + \Delta v)} \right] \{ e^{-(v-(v_0+\Delta v))^2/v_0^2} - e^{-(v+(v_0+\Delta v))^2/v_0^2} \} \]

  - Simple Maxwellian velocity distribution
    \[ f_{1,\text{Gau}}(v) = \frac{4}{\sqrt{\pi}} \left( \frac{v^2}{v_0^3} \right) e^{-v^2/v_0^2} \]

  - "Modified" simple Maxwellian velocity distribution
    \[ f_{1,\text{Gau},k}(v) = \frac{v^2}{N_{f,k}} \left( e^{-v^2/kv_0^2} - e^{-v_{\text{max}}^2/kv_0^2} \right)^k \quad \text{for} \ v \leq v_{\text{max}} \]
Numerical results

- Reconstructed $f_{1,\text{Bayesian}}(v)$ with the input WIMP mass
  
  ($^{76}\text{Ge}$, 2 - 50 keV, 500 events, $m_\chi = 25$ GeV, $f_{1,\text{sh},v_0}(v) \Rightarrow f_{1,\text{sh},v_0}(v)$, flat-dist.)

[CLS, IJMPD 24, 1550090 (2015)]
Numerical results

- Reconstructed $f_{1,\text{Bayesian}}(v)$ with the input WIMP mass
  
  $^{76}\text{Ge}$, 5 - 50 keV, 500 events, $m_\chi = 25$ GeV, $f_{1,\text{sh},v_0}(v) \Rightarrow f_{1,\text{sh},v_0}(v)$, flat-dist.)
Determination of the WIMP mass
Determination of the WIMP mass

- Estimating the moments of the WIMP velocity distribution

\[ \langle v^n \rangle = \alpha^n \left[ \frac{2Q_{\min}^{1/2}r_{\min}}{F^2(Q_{\min})} + l_0 \right]^{-1} \left[ \frac{2Q_{\min}^{(n+1)/2}r_{\min}}{F^2(Q_{\min})} + (n + 1)l_n \right] \]

\[ l_n = \sum_a \frac{Q_a^{(n-1)/2}}{F^2(Q_a)} \]

\[ r_{\min} = \left( \frac{dR}{dQ} \right)_{\text{expt, } Q=Q_{\min}} = r_1 e^{k_1(Q_{\min} - Q_s, 1)} \]

- Determining the WIMP mass

\[ m_\chi |_{\langle v^n \rangle} = \frac{\sqrt{m_X m_Y} - m_\chi \mathcal{R}_n}{\mathcal{R}_n - \sqrt{m_X/m_Y}} \]

\[ \mathcal{R}_n = \left[ \frac{2Q_{\min,X}^{(n+1)/2}r_{\min,X}/F_X^2(Q_{\min,X}) + (n + 1)l_n,X}{2Q_{\min,X}^{1/2}r_{\min,X}/F_X^2(Q_{\min,X}) + l_0,X} \right]^{1/n} (X \rightarrow Y)^{-1} (n \neq 0) \]

- Assuming a dominant SI scalar WIMP-nucleus interaction

\[ m_\chi |_{\sigma} = \frac{(m_\chi/m_Y)^{5/2} m_Y - m_\chi \mathcal{R}_\sigma}{\mathcal{R}_\sigma - (m_\chi/m_Y)^{5/2}} \]

\[ \mathcal{R}_\sigma = \frac{\mathcal{E}_Y}{\mathcal{E}_X} \left[ \frac{2Q_{\min,Y}^{1/2}r_{\min,Y}/F_Y^2(Q_{\min,Y}) + l_0,Y}{2Q_{\min,Y}^{1/2}r_{\min,Y}/F_Y^2(Q_{\min,Y}) + l_0,Y} \right] \]

[CLS and M. Drees, arXiv:0710.4296]

[M. Drees and CLS, JCAP 0706, 011 (2007)]

[C. L. Shan (XAO-CAS)]

SYSU, September 17, 2015 p. 26
Determination of the WIMP mass

- Reconstructed $m_{\chi,\text{rec}}$
  $(^{28}\text{Si} + ^{76}\text{Ge}, Q_{\text{max}} < 100 \text{ keV}, 2 \times 50 \text{ events})$

[M. Drees and CLS, JCAP 0806, 012 (2008)]
Determinations of the WIMP-nucleon couplings
Estimation of the SI scalar WIMP-nucleon coupling
Estimation of the SI scalar WIMP-nucleon coupling

- **Spin-independent (SI) scalar** WIMP-nucleus cross section
  
  \[ \sigma_{SI}^0 = \left( \frac{4}{\pi} \right) m_{r,N}^2 \left[ Z f_p + (A - Z) f_n \right]^2 \approx \left( \frac{4}{\pi} \right) m_{r,N}^2 A^2 |f_p|^2 = A^2 \left( \frac{m_{r,N}}{m_{r,p}} \right)^2 \sigma_{\chi p}^{SI} \]

  \[ \sigma_{\chi p}^{SI} = \left( \frac{4}{\pi} \right) m_{r,p}^2 |f_p|^2 \]

  \( f_{(p,n)} \): effective SI scalar WIMP-proton/neutron couplings

- Rewriting the integral over \( f_1(\nu)/\nu \)

  \[ \left( \frac{dR}{dQ} \right)_{\text{expt, } Q=Q_{\min}} = \frac{\mathcal{E} \rho_0 A^2}{2m_{\chi} m_{r,p}^2} \left[ \left( \frac{4}{\pi} \right) m_{r,p}^2 |f_p|^2 \right] F^2(Q_{\min}) \left\{ m_{r,N} \sqrt{\frac{2}{m_N}} \left[ \frac{2Q_{\min}^{1/2} r_{\min}^Z}{F^2(Q_{\min})} + l_0 \right]^{-1} \left[ \frac{2 r_{\min}^Z}{F^2(Q_{\min})} \right] \right\} \]

- Estimating the **SI scalar WIMP-nucleon coupling**

  \[ |f_p|^2 = \frac{1}{\rho_0} \left[ \frac{\pi}{4\sqrt{2}} \left( \frac{1}{\mathcal{E} Z A_Z^2 \sqrt{m_Z}} \right) \right] \left[ \frac{2Q_{\min,Z}^{1/2} r_{\min,Z}^Z}{F_Z^2(Q_{\min,Z})} + l_{0,Z} \right] (m_{\chi} + m_Z) \]

Estimation of the SI scalar WIMP-nucleon coupling

- Reconstructed $|f_p|^2_{\text{rec}}$
  - $^{76}\text{Ge} (+^{28}\text{Si} + ^{76}\text{Ge})$, $Q_{\text{max}} < 100$ keV, $\sigma_{\chi p}^{\text{SI}} = 10^{-8}$ pb, $1(3) \times 50$ events

[CLS, arXiv:1103.0481]
Estimation of the SI scalar WIMP-nucleon coupling

- Reconstructed $|f_p|_{\text{rec}}^2$ vs. reconstructed $m_{\chi,\text{rec}}$

$(^{76}\text{Ge} + ^{28}\text{Si} + ^{76}\text{Ge}), Q_{\text{max}} < 100 \text{ keV}, \sigma_{\chi p}^{\text{SI}} = 10^{-8} \text{ pb}, 1(3) \times 50 \text{ events}$

[CLS, arXiv:1103.0481]
Determinations of ratios of WIMP-nucleon cross sections
Determination of the ratio of SD WIMP-nucleon couplings

- **Spin-dependent (SD) axial-vector** WIMP-nucleus cross section

\[
\sigma^{\text{SD}}_0 = \left( \frac{32}{\pi} \right) G_F^2 m_{r,N}^2 \left( \frac{J + 1}{J} \right) \left[ \langle S_p \rangle a_p + \langle S_n \rangle a_n \right]^2
\]

\[
\sigma^{\text{SD}}_{\chi_{p/n}} = \left( \frac{32}{\pi} \right) G_F^2 m_{r,p/n}^2 \cdot \left( \frac{3}{4} \right) a_{p/n}^2
\]

- Total nuclear spin

- \( \langle S_{(p,n)} \rangle \): expectation values of the proton/neutron group spin

- \( a_{(p,n)} \): effective SD axial-vector WIMP-proton/neutron couplings

- **Determining the ratio of two SD axial-vector WIMP-nucleon couplings**

\[
\left( \frac{a_n}{a_p} \right)^{\text{SD}}_{\pm,n} = -\frac{\langle S_p \rangle X \pm \langle S_p \rangle Y \mathcal{R}_{J,n}}{\langle S_n \rangle X \pm \langle S_n \rangle Y \mathcal{R}_{J,n}} \mathcal{R}_{J,n}^{-1/2}
\]

\[
\mathcal{R}_{J,n} = \left[ \left( \frac{J_X}{J_X + 1} \right) \left( \frac{J_Y + 1}{J_Y} \right) \frac{\mathcal{R}_\sigma}{\mathcal{R}_n} \right]^{1/2} \quad (n \neq 0)
\]

[M. Drees and CLS, arXiv:0903.3300]
Determination of the ratio of SD WIMP-nucleon couplings

- Reconstructed \((a_n/a_p)^{SD}_{\text{rec,1}}\)

\[ (^{73}\text{Ge} + ^{37}\text{Cl} \text{ and } ^{19}\text{F} + ^{127}\text{I}, \quad Q_{\text{min}} > 5 \text{ keV}, \quad Q_{\text{max}} < 100 \text{ keV}, \quad 2 \times 50 \text{ events, } \quad m_\chi = 100 \text{ GeV}) \]
Determinations of ratios of WIMP-nucleon cross sections

- Differential rate for combined SI and SD cross sections

\[
\left( \frac{dR}{dQ} \right)_{\text{expt}, Q=Q_{\text{min}}} = \mathcal{E} \left( \frac{\rho_0 \sigma_{0}^{\text{SI}}}{2m_\chi m_r^2} \right) \left[ F_{\text{SI}}^2(Q) + \left( \frac{\sigma_{\chi p}^{\text{SD}}}{\sigma_{\chi p}^{\text{SI}}} \right) c_p F_{\text{SD}}^2(Q) \right] \int_{v_{\text{min}}}^{v_{\text{max}}} \left[ \frac{f_1(v)}{v} \right] dv
\]

\[c_p \equiv \frac{4}{3} \left( \frac{J + 1}{J} \right) \left[ \langle S_p \rangle + \left( \frac{a_n/a_p}{A} \right) \langle S_n \rangle \right]^2\]

- Determining the ratio of two WIMP-proton cross sections

\[
\frac{\sigma_{\chi p}^{\text{SD}}}{\sigma_{\chi p}^{\text{SI}}} = \frac{F_{\text{SI}, Y}^2(Q_{\text{min}, Y}) \mathcal{R}_{m,XY} - F_{\text{SI}, X}^2(Q_{\text{min}, X})}{c_p, X F_{\text{SD}, X}^2(Q_{\text{min}, X}) - c_p, Y F_{\text{SD}, Y}^2(Q_{\text{min}, Y}) \mathcal{R}_{m,XY}}
\]

\[
\mathcal{R}_{m,XY} \equiv \left( \frac{r_{\text{min}, X}}{\mathcal{E}_X} \right) \left( \frac{c_Y}{r_{\text{min}, Y}} \right) \left( \frac{m_Y}{m_X} \right)^2
\]

- Determining the ratio of two SD axial-vector WIMP-nucleon couplings

\[
\left( \frac{a_n}{a_p} \right)^{\text{SI+SD}} = \left( c_p, X s_{n/p, X} - c_p, Y s_{n/p, Y} \right) \pm \sqrt{c_p, X c_p, Y} \left| s_{n/p, X} - s_{n/p, Y} \right|
\]

\[
c_p, X \equiv \frac{4}{3} \left( \frac{J_X + 1}{J_X} \right) \left[ \frac{\langle S_p \rangle X}{A_X} \right]^2 \left[ F_{\text{SI}, Z}^2(Q_{\text{min}, Z}) \mathcal{R}_{m, YZ} - F_{\text{SI}, Y}^2(Q_{\text{min}, Y}) \right] F_{\text{SD}, X}^2(Q_{\text{min}, X})
\]

[M. Drees and CLS, arXiv:0903.3300]
Determinations of ratios of WIMP-nucleon cross sections

- Reconstructed \( (a_n/a_p)^{\text{SI}+\text{SD}}_{\text{rec}} \) vs. \( (a_n/a_p)^{\text{SD}}_{\text{rec},1} \)

\( ^{19}\text{F} + ^{127}\text{I} + ^{28}\text{Si}, \) \( Q_{\text{min}} > 5 \text{ keV}, \) \( Q_{\text{max}} < 100 \text{ keV}, \) \( 3 \times 50 \) events, \( \sigma_{\chi p}^{\text{SI}} = 10^{-8}/10^{-10} \) pb, \( a_p = 0.1, \) \( m_\chi = 100 \text{ GeV} \)

[CLS, JCAP 1107, 005 (2011)]
Determinations of ratios of WIMP-nucleon cross sections

- Reconstructed \( \left( \frac{\sigma_{\chi p}^{SD}}{\sigma_{\chi p}^{SI}} \right)_{\text{rec}} \) and \( \left( \frac{\sigma_{\chi n}^{SD}}{\sigma_{\chi p}^{SI}} \right)_{\text{rec}} \)

\((^{19}\text{F} + ^{127}\text{I} + ^{28}\text{Si} \text{ vs.} \ ^{23}\text{Na}/^{131}\text{Xe} + ^{76}\text{Ge}, \ Q_{\text{min}} > 5 \text{ keV}, \ Q_{\text{max}} < 100 \text{ keV}, \ \sigma_{\chi p}^{SI} = 10^{-8} \text{ pb}, \ a_{p} = 0.1, \ m_{\chi} = 100 \text{ GeV}, \ 3/2 \times 50 \text{ events})\)

[CLS, JCAP 1107, 005 (2011)]
Summary
Summary

- Once two or more experiments with different target nuclei observe positive WIMP signals, we could reconstruct:
  - WIMP mass $m_\chi$
  - 1-D velocity distribution $f_1(v)$
  - SI WIMP-proton coupling $|f_p|^2$
  - ratio between the SD WIMP-nucleon couplings $a_n/a_p$
  - ratios between the SD and SI WIMP-nucleon cross sections $\sigma^{SD}_{\chi(p,n)}/\sigma^{SI}_{\chi p}$

- For these analyses the local density, the velocity distribution, and the mass/couplings on nucleons of halo WIMPs are not required priorly.

- For a WIMP mass of $\mathcal{O}(100 \text{ GeV})$, with only $\mathcal{O}(50)$ events from one experiment and less than $\sim 20\%$ unrejected backgrounds, these quantities could be estimated with statistical uncertainties of 10\% - 40\%.
Thank you very much for your attention!