Improvement of the Determination of the WIMP Mass from Direct Dark Matter Detection Data

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based on JCAP 0806, 012 (arXiv:0803.4477 [hep-ph])
Determining the WIMP mass model-independently

Correcting the systematic deviation of the reconstructed WIMP mass

Summary
Determining the WIMP mass model-independently

- Differential event rate for elastic WIMP-nucleus scattering

\[
\frac{dR}{dQ} = A F^2(Q) \int_{\nu_{\text{min}}}^{\nu_{\text{esc}}} \frac{f_1(\nu)}{\nu} d\nu
\]

Here

\[
\nu_{\text{min}} = \alpha \sqrt{Q}
\]

is the minimal incoming velocity of incident WIMPs that can deposit the recoil energy \(Q\) in the detector.

\[
A \equiv \frac{\rho_0 \sigma_0}{2 m_\chi m_{r,N}^2} \quad \alpha \equiv \sqrt{\frac{m_N}{2 m_{r,N}^2}} \quad m_{r,N} = \frac{m_\chi m_N}{m_\chi + m_N}
\]

\(\rho_0\): WIMP density near the Earth

\(\sigma_0\): total cross section ignoring the form factor suppression

\(F(Q)\): elastic nuclear form factor

\(f_1(\nu)\): one-dimensional velocity distribution of halo WIMPs
Determining the WIMP mass model-independently

- Differential event rate for elastic WIMP-nucleus scattering

\[
\frac{dR}{dQ} = A F^2(Q) \int_{v_{\text{min}}}^{v_{\text{esc}}} \left[ \frac{f_1(v)}{v} \right] dv
\]

Here

\[v_{\text{min}} = \alpha \sqrt{Q}\]

is the minimal incoming velocity of incident WIMPs that can deposit the recoil energy \(Q\) in the detector.

\[A \equiv \frac{\rho_0 \sigma_0}{2 m_\chi m_{r,N}^2}\]

\[\alpha \equiv \sqrt{\frac{m_N}{2 m_{r,N}^2}}\]

\[m_{r,N} = \frac{m_\chi m_N}{m_\chi + m_N}\]

\(\rho_0\): WIMP density near the Earth
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\(F(Q)\): elastic nuclear form factor
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Determining the WIMP mass model-independently

- Determining the moments of the WIMP velocity distribution

\[
\langle v^n \rangle = \alpha^n \left[ \frac{2 Q_{\text{thre}}^{1/2} r_{\text{thre}}}{F^2(Q_{\text{thre}})} + l_0 \right]^{-1} \left[ \frac{2 Q_{\text{thre}}^{(n+1)/2} r_{\text{thre}}}{F^2(Q_{\text{thre}})} + (n + 1) l_n \right]
\]

\[
l_n = \sum_a Q_a^{(n-1)/2} F_a^2(Q_{\text{thre}})
\]

\[
r_{\text{thre}} = \left( \frac{dR}{dQ} \right)_{Q=Q_{\text{thre}}}
\]

- Determining the WIMP mass

\[
m_\chi = \sqrt{m_X m_Y - m_X R_n} \frac{R_n}{R_n - \sqrt{m_X/m_Y}}
\]

\[
R_n = \frac{\alpha_Y}{\alpha_X}
\]

\[
= \left[ \frac{2 Q_{\text{thre}, X}^{(n+1)/2} r_{\text{thre}, X}/F_X^2(Q_{\text{thre}, X}) + (n + 1) l_{n, X}}{2 Q_{\text{thre}, X}^{1/2} r_{\text{thre}, X}/F_X^2(Q_{\text{thre}, X}) + l_{0, X}} \right]^{1/n} \left( X \rightarrow Y \right)^{-1} (n \neq 0)
\]

[C. L. Shan, SUSY08, Seoul, Korea]
Determining the WIMP mass model-independently

- **Spin-independent (SI) WIMP-nucleus cross section** *(neutralino)*
  \[
  \sigma_0^{\text{SI}} \simeq \left(\frac{4}{\pi}\right) m_{r,N} A^2 |f_p|^2
  \]
  \(f_p\): effective WIMP – proton coupling

- **Determining the WIMP mass**
  \[
  m_\chi^{\text{SI}} = \frac{(m_X/m_Y)^{5/2} m_Y - m_X \mathcal{R}_\sigma}{\mathcal{R}_\sigma - (m_X/m_Y)^{5/2}}
  \]
  \[
  \mathcal{R}_\sigma = \frac{\mathcal{E}_Y}{\mathcal{E}_X} \left[\frac{2 Q_{\text{thre},X}^1 r_{\text{thre},X} / F_X^2 (Q_{\text{thre},X}) + l_{0,X}}{2 Q_{\text{thre},Y}^1 r_{\text{thre},Y} / F_Y^2 (Q_{\text{thre},Y}) + l_{0,Y}}\right]
  \]

- **Neglecting** \(Q_{\text{thre}}\)
  \[
  \mathcal{R}_\sigma = \frac{\mathcal{E}_Y}{\mathcal{E}_X} \left(\frac{l_{0,X}}{l_{0,Y}}\right)
  \]
  \[
  \sigma \left( m_\chi^{\text{SI}} \right) = \frac{\mathcal{R}_\sigma \left( m_X/m_Y \right)^{5/2} |m_X - m_Y|}{\mathcal{R}_\sigma - (m_X/m_Y)^{5/2}} \left[\frac{\sigma^2 \left( l_{0,X} \right)}{l_{0,X}^2} + \frac{\sigma^2 \left( l_{0,Y} \right)}{l_{0,Y}^2}\right]^{1/2}
  \]

[M. Drees and CLS, JCAP 0806, 012]
Determining the WIMP mass model-independently

- Reconstructed WIMP mass $m_{\chi}(n = 1)$ and $m_{\chi}^{SI}$
  
  $(1 - 200 \text{ keV}, \text{^{76}Ge + ^{28}Si}, 25 + 25 / 50 + 50 \text{ events})$

[C.L. Shan, SUSY08, Seoul, Korea p. 6]
Correcting the systematic deviation of the reconstructed WIMP mass

- Matching $Q_{\text{max}}$

$$Q_{\text{max}, \text{Si}} = \left( \frac{\alpha_{\text{Ge}}}{\alpha_{\text{Si}}} \right)^2 Q_{\text{max}, \text{Ge}} \iff v_{\text{max}} = \alpha \sqrt{Q_{\text{max}}}$$

- Algorithmic $\chi^2$ fit

$$\chi^2 = \sum_{i,j} (f_{i,x} - f_{i,y}) C_{ij}^{-1} (f_{j,x} - f_{j,y})$$

where

$$f_{i,x} = \alpha_X^i \left[ \frac{2Q_{\text{min},x}^{(i+1)/2} r_x (Q_{\text{min},x})/F_X^2 (Q_{\text{min},x}) + (i + 1) l_{i,x}}{2Q_{\text{min},x}^{1/2} r_x (Q_{\text{min},x})/F_X^2 (Q_{\text{min},x}) + l_{0,x}} \right] \left( \frac{1}{300 \text{ km/s}} \right)^i$$

$$f_{n_{\text{max}}+1,x} = \mathcal{E}_X \left[ \frac{A_X^2}{2Q_{\text{min},x}^{1/2} r_x (Q_{\text{min},x})/F_X^2 (Q_{\text{min},x}) + l_{0,x}} \right] \left( \frac{\sqrt{m_X}}{m_X + m_X} \right)$$

$$C_{ij} = \text{cov} (f_{i,x}, f_{j,x}) + \text{cov} (f_{i,y}, f_{j,y})$$

[M. Drees and CLS, JCAP 0806, 012]
Improvement of the Determination of the WIMP Mass from Direct DM Detection Data

Correcting the systematic deviation of the reconstructed WIMP mass

- Reconstructed WIMP mass
  \( Q_{\text{max}} < 100 \text{ keV}, \ 76\text{Ge} + 28\text{Si}, \ 50 + 50 \text{ events} \)

[Plot showing the relationship between the reconstructed WIMP mass and the input mass, with different matching scenarios.

[M. Drees and CLS, JCAP 0806, 012]

C. L. Shan, SUSY08, Seoul, Korea
Correcting the systematic deviation of the reconstructed WIMP mass

- Reconstructed WIMP mass
  \( Q_{\text{max}} < 100 \text{ keV}, ^{76}\text{Ge} + ^{28}\text{Si}, 50 + 50 \text{ events, late infall compo.} \)

![Graph showing the correction of the systematic deviation of the reconstructed WIMP mass.](image)

\[ \text{[M. Drees and CLS, JCAP 0806, 012]} \]
Correcting the systematic deviation of the reconstructed WIMP mass

- Reconstructed WIMP mass
  \( (Q_{\text{max}} < 100 \text{ keV}, \text{ } ^{76}\text{Ge} + ^{28}\text{Si}, 500 + 500 \text{ events}) \)

\[ m_{\chi, \text{ge, li, lo}} \text{[GeV]} \]

\[ m_{\chi, \text{in}} \text{[GeV]} \]

500 + 500 events, Si and Ge, standard halo, \( Q_{\text{max}} < 100 \text{ keV} \)

[M. Drees and CLS, JCAP 0806, 012]
Correcting the systematic deviation of the reconstructed WIMP mass

- **Reconstructed WIMP mass**
  
  \[ Q_{\text{max}} < 75 \text{ keV}, \ 40\text{Ar} + 136\text{Xe}, \ 50 + 50 \text{ events} \]

  \[ \text{[M. Drees and CLS, JCAP 0806, 012]} \]
Summary

- Once two or more experiments with different detector materials obtain positive WIMP signals, we can determine the WIMP mass.

- Our methods are model-independent and require only measured recoil energies.

- The deviation of the reconstructed WIMP mass from the true one can be corrected by matching more suitable cut-off energies.

- With $\sim 100 \text{ keV}$ maximal measuring energies and $2 \times 50$ events, a WIMP mass of 50 GeV can be estimated with an error of $\sim 35\%$. 