Introduction of LDA

Last Class:
1. Matlab Programming
2. An example of PIV in Matlab code
3. EDPIV
4. PIV plugin for ImageJ
5. JPIV

Today’s Contents:
1. Fundamentals of LDA
2. Turbulent Flow Properties
3. LDA Principle : Laser Optics
4. The Doppler Effect
5. Gaussian Beam and Measurement Volume
Traditional Methods of Flow Measurements

Hot-wire anemometers (HWA)

Pitot tubes

Turbine meter
Fundamentals of LDA

Since the first LDA measurement was successfully tested by Yeh and Cummins (1964), the method is undergoing continuous development.
Fundamentals of LDA (Cont.)
Some Effects of LDA

(A) **Velocity bias effect:** The velocity bias arises from the effect that the velocity sampling rate in LDA measurements is not time-equidistant but depends on the velocity itself. In concrete terms, high velocities are proportionally more frequently sampled than low velocities.

(B) **Fringe distortion:** As an optical phenomenon, the fringe distortion in the LDA measurement volume is mainly caused by improper optical layout or by irregular i.e. asymmetrical laser beam refractions on the medium interface.

(C) **Spatial velocity gradient effect:** For flows with spatial velocity gradient, such as the flows in the turbulent boundary layers, LDA measurements suffer from measurement errors in both mean and fluctuation velocities.

(D) **Non-stationary flow measurements:** In the enforced non-stationary turbulent flows, flow fluctuations comprise both the enforced velocity variation because of the flow periodicity, for instance, and the stochastic velocity fluctuations because of the flow turbulence.
Obviously the LDA technique with respect to its applications and application optimizations still needs to be developed.

(A) *Optical aberration and astigmatism:* In LDA applications, the optical aberrations generally exist in each measurement of internal flows, where the laser beams must transmit through at least one optical window and hence undergo refractions.

(B) *Three-component flow measurements in circular pipes:* Another most common case of LDA measurements is referred to flow measurements in circular pipes. In this case, the optical aberration associated with the laser beam refractions is much more complex and serious than that at a plane surface.
(C) **Dual Measurement Method (DMM):** As is known, LDA measurements are measurements of velocity components. Because of this property, there are sometimes difficulties in accurately resolving a component, say that one in the secondary flow, which is much smaller against the other components.

(D) **Zero Correlation Method (ZCM):** In measuring the turbulent flows, it is often required to measure both the mean velocity and the turbulence quantities such as the turbulence intensity and turbulent stresses in the Reynolds stress matrix.
Turbulent Flow Properties

Turbulent flows are known as the flows with irregular fluctuations of fluid particles in motion. To describe the turbulent flow with velocity fluctuations, the flow velocity is usually split into a time-averaged mean and a fluctuation velocity.

\[ u(t) = \bar{u} + u'(t) \]

In most cases dealing with stationary turbulent flows, flow fluctuations are stochastic and can be approximated by the Gaussian probability density function as given by

\[ \sigma_u = \sqrt{\frac{1}{T} \int_0^T u'^2 dt} \]

\[ p_{df_u} = \frac{1}{\sqrt{2\pi} \sigma_u} e^{-\frac{(u-\bar{u})^2}{2\sigma_u^2}} \]
Gaussian Probability Function

In the figure the corresponding probability density function has been shown to approximate the distribution of fluctuation velocities. The probability of random velocities occurring in the range \( u = u \pm \sigma_u \) is calculated as

\[
P(\sigma_u) = \int_{\bar{u}-\sigma_u}^{\bar{u}+\sigma_u} pdf_u\,du = \frac{2}{\sqrt{\pi}} \int_{0}^{1/\sqrt{2}} e^{-z^2} dz = \text{erf} \left( \frac{1}{\sqrt{2}} \right) \approx 68.3\%
\]
Isotropic and Anisotropic Turbulences

Since velocity fluctuations in each turbulent flow are always three-dimensional, it distinguishes between isotropic and anisotropic turbulences. Local isotropy of turbulence is given at large Reynolds numbers and conditions without remarkable influences of any boundaries. The asymmetrical distribution of velocity fluctuations around the mean velocity vector has usually been found in the flows where large velocity gradients are present.
LDA Principle : Laser optics

The technique of Laser Doppler Anemometry (LDA), as the name stands for, is a technique of using the laser light and the Doppler effect for velocity measurements. It is an optical method and hence tightly related to both the physical and geometrical optics.

By disregarding the light polarization the spatial propagation of a plane light wave of the amplitude $E_0$ in the positive x-direction can be expressed by

$$E = E_0 \cos(\omega t - kx) \quad k = 2\pi/\lambda.$$

$$\frac{d(\omega t - kx)}{dt} = 0$$

$$c = \frac{dx}{dt} = \frac{\omega}{k}$$
Light Propagation in Medium

Since the light frequency does not change while the light is refracted at the interface between two mediums, The change of light speed from one medium \((n_1)\) into another \((n_2)\) can be written with respect to \(c = \nu \lambda\) as

\[
\frac{c_2}{c_1} = \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1}
\]

Snell’s law

\[
n_1 \sin \epsilon_1 = n_2 \sin \epsilon_2
\]
The Doppler Effect

The Doppler effect in optics is associated with the light propagation and accounts for the frequency shift when the light source is moving or light is reflected off a moving surface.

Color changes depending on the object’s moving direction.

http://glasnost.itcarlow.ie/~powerk/audio/environmentalaudio.htm

http://knol.google.com/k/stephane-jourdan/the-doppler-effect-s-contribution-n/3dfvm2oyvur0n/11#
The Doppler Effect

Because the light velocity is independent of the motion of the light source, the time used for wave transmission through the path $s$ is still equal to $t$.

\[ \frac{ct}{\lambda_0} = \frac{ct - (\vec{u}_s \cdot \vec{l}) t}{\lambda_1} \]

\[ \lambda_1 = \left(1 - \frac{\vec{u}_s \cdot \vec{l}}{c}\right)\lambda_0 \]

The number of waves is the same for both cases.
The Doppler Effect

With respect to the constant light speed given by \( \lambda_1 \nu_1 = \lambda_0 \nu_0 = c \), the frequency of the light wave is calculated as

\[
\nu_1 = \frac{\nu_0}{1 - \frac{u_s \cdot l}{c}}
\]

Because of \( u_s \cdot l/c \ll 1 \) the above equation is simplified to

or when the moving velocity of the receiver is set by \( u_r = -u_s \)
The Doppler Effect of LDV: Single Beam

In relying on LDA principles, a scattering system is considered to consist of a fixed light source, a moving object (i.e. a small particle) and a fixed observer to receive the light scattered by the moving particle. The corresponding optical arrangement has been illustrated in the following figure.

\[ v_2 = v_1 \left( 1 + \frac{\vec{u}_p \cdot \vec{l}_2}{c} \right) \]

\[ v_2 = v_0 \left( 1 - \frac{\vec{u}_p \cdot \vec{l}_1}{c} \right) \left( 1 + \frac{\vec{u}_p \cdot \vec{l}_2}{c} \right) \]
The Doppler Effect of LDV: Single Beam

Because the flow velocity is always negligible against the light speed as given by $u_p \cdot l_1/c << 1$ and $u_p \cdot l_2/c << 1$,

The shifted frequency $\nu_2$ is in the order of $\nu_0$ and therefore still too high to be measured by conventional devices that are found in usual laboratories.
The Doppler Effect of LDA: Dual Beams

The dual beam configuration of LDA optics relies upon the superposition of two light waves that are differently shifted by the Doppler effect.

\[
E_a = E_{a0} \cos(\omega_a t - k_a x) \\
E_b = E_{b0} \cos(\omega_b t - k_b x)
\]

For the case \( E_{b0} = 1.5E_{a0} \) and \( \omega_b = 1.1\omega_a, \ k_b = 1.1k_a \).
Light Coherence (1)

Coherence describes all properties of the correlation between physical quantities of a wave.

**Temporal coherence** is the measure of the average correlation between the value of a wave and itself delayed by $\tau$.

A wave containing only a single frequency (monochromatic) is perfectly correlated at all times according to the above relation.

Conversely, a wave whose phase drifts quickly (chromatic) will have a short coherence time.
Spatial coherence is the cross-correlation between two points in a wave for all times.

Ex: laser (a few cm~ a few meters)

Ex: white light (<<1 cm)
The Doppler Effect of LDA: Dual Beams

The superposition of these two waves is simply given as

\[ E = E_{a0} \cos(\omega_a t - k_a x) + E_{b0} \cos(\omega_b t - k_b x) \]

Obviously the superimposed wave possesses both a high frequency and a low modulation frequency. For calculating these two frequencies Eq. (3.23) is rearranged as

\[ E = E_{a0} [\cos(\omega_a t - k_a x) + \cos(\omega_b t - k_b x)] + (E_{b0} - E_{a0}) \cos(\omega_b t - k_b x) \]
Beat Frequency

By applying the trigonometric identity

\[
\cos \alpha + \cos \beta = 2 \cos \frac{1}{2} (\alpha + \beta) \cos \frac{1}{2} (\alpha - \beta)
\]

\[
E = (E_{a0} + E_{b0}) \cos (\bar{\omega} t - \bar{k} x) \cdot \cos (\omega_m t - k_m x) \\
+ (E_{b0} - E_{a0}) \sin (\bar{\omega} t - \bar{k} x) \cdot \sin (\omega_m t - k_m x)
\]

where \( \bar{\omega} = \frac{1}{2} (\omega_a + \omega_b), \quad \omega_m = \frac{1}{2} (\omega_a - \omega_b) \) and \( \bar{k} = \frac{1}{2} (k_a + k_b), \quad k_m = \frac{1}{2} (k_a - k_b) \)

The amplitude of the high frequency oscillation is given by the modulated wave intensities of the superimposed wave

\[
E_m = (E_{a0} + E_{b0}) \cos (\omega_m t - k_m x) \\
E_m^2 = (E_{a0} + E_{b0})^2 \cos^2 (\omega_m t - k_m x)
\]

\( E_m^2 \) oscillates with an angular frequency of \( 2\omega_m = \omega_a - \omega_b \) which is known as the beat frequency.
Fringe Visibility

\[
\eta = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}
\]
LDA Frequency

\[ 2\omega_m = \omega_a - \omega_b \]

\[
\nu_{2a} \approx \nu_0 \left( 1 - \frac{\vec{u}_p \cdot \vec{l}_{1a}}{c} + \frac{\vec{u}_p \cdot \vec{l}_2}{c} \right) \quad \nu_{2b} \approx \nu_0 \left( 1 - \frac{\vec{u}_p \cdot \vec{l}_{1b}}{c} + \frac{\vec{u}_p \cdot \vec{l}_2}{c} \right)
\]

\[
\nu_D = |\nu_{2a} - \nu_{2b}| = \frac{\nu_0}{c} \left| \vec{u}_p \cdot \left( \vec{l}_{1b} - \vec{l}_{1a} \right) \right|
\]

Because of \( c/\nu_0 = \lambda_0 \) and \( |\vec{u}_p \cdot \left( \vec{l}_{1b} - \vec{l}_{1a} \right)| = 2u_{p,\perp} \sin \alpha \)
Fringe Model on the Interference

The fringe model presented above is a very useful tool to understand the LDA measurement principle. It also represents a very convenient means to make further studies of diverse optical phenomena influencing the measurement accuracy.
Optical Polarization

For interference, the orientations of both polarized light beams must not be orthogonal!!

![Diagram of optical polarization](image)

In Figure 1, light passing through crossed polarizers shows that vertically polarized light waves are eliminated.
Fringes
Flow Direction: Frequency Shift

An ambiguity to the flow direction of the particle, however, exists because a positive and a negative velocity of the same magnitude will cause the same Doppler frequency. In order to resolve the flow direction from each Doppler burst, the technique of using Bragg cells to slightly shift the frequency in one or both of two laser beams in each laser beam pair has become a standard.
Gaussian Beam Properties

$$I(r) = I_0 e^{-2(r/w)^2}$$

the Rayleigh length

$$R = z \left[ 1 + \left( \frac{\pi w_0^2}{\lambda z} \right)^2 \right]$$

$$\frac{dR}{dz} = 0 \quad z_R = \frac{\pi \cdot w_0^2}{\lambda}$$

$$w = w_0 \sqrt{1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2}$$

$$R_{\text{min}} = 2z_R = \frac{2\pi \cdot w_0^2}{\lambda}$$

$$\theta = \frac{w_0}{z_R}$$
Transmission of Gaussian Beam

\[ w_0 = \frac{f}{\sqrt{(s' - f)^2 + z_R^2}} w_0' \]

\[ s = f + \frac{f^2 (s' - f)}{(s' - f)^2 + z_R^2} \]
Measurement Volume Size

plane wave front at the laser beam waist enables one to create uniform fringes in the measurement volume and hence to enhance the reliability and accuracy of measurements. Otherwise fringe distortion in the measurement volume will occur and lead to measurement errors.

In general, $d_{mv}$ is in the order of about 0.05–0.1 mm and $2a$ is about 0.5–3 mm.

The number of fringes