Post-Processing of PIV Data

Last Class:
1. Digital PIV Evaluation (FFT-based CC, Direct CC)
2. Correlation Signal Enhancement
3. Advanced Interrogation Techniques
4. Peak Detection & displacement Estimation
5. Measurement Noise and Accuracy

Today’s Contents:
1. Data Validation
2. Vector Field Operator (Differentials & Integrals)
3. Standard Differential Scheme
4. Implementation of Differential & Integral quantities with PIV data
5. 3P PIV Measurements
Data Validation

It shows the instantaneous flow field above a NACA 0012 airfoil at a free stream Mach number $Ma = 0.75$. 
A displacement vector can then be classified as questionable when it is ‘conflicting’ with at least half its neighbors.
PIV data validation by means of median filtering has been proposed by Westerweel. While median filtering is frequently utilized in image processing to remove spurious noise, it may also be used for the efficient treatment of spurious velocity vectors. Median filtering simply speaking means that all neighboring velocity vectors $\mathbf{U}_{2D}(n)$ are sorted linearly either with respect to the magnitude of the velocity vector, or their $U$ and $V$ components. The central value in this order (i.e. either the fourth or fifth of eight neighbors) is the median value. The velocity vector under inspection $\mathbf{U}_{2D}(i, j)$ is considered valid if

$$|\mathbf{U}_{2D}^{\text{med}} - \mathbf{U}_{2D}(i, j)| < \epsilon_{\text{thresh}}$$
Other Validation Filters

Minimum correlation filter
As mentioned earlier, a low correlation coefficient is indicative of a strong loss a particle match and may have a variety of causes. Thus, a validation filter may be very helpful in detecting problematic areas in the field of view. However it is of lesser importance for the actual validation of PIV data, as low correlation values do not necessarily point to invalid displacement readings.

Signal-to-noise filter
Here the signal-to-noise ratio in the correlation plane – defined as the quotient of correlation peak height with respect to the mean correlation level – is used to validate the data. However its use is questionable because mismatched particle images or stationary background features can also produce high levels of correlation.
Replacement Schemes

After having validated all PIV data it is possible to fill in missing data using, for instance, bilinear interpolation. Some post-processing methods also require smoothing of the data. The reason is that the experimental data is affected by noise in contrast to numerical data. A simple convolution of the data with a $2 \times 2$, $3 \times 3$ or larger smoothing kernel (with equal weights) is generally sufficient for this purpose.

High-quality PIV data typically exhibits less than 1% of spurious vectors under regular conditions and less than 5% in rather challenging experimental situations.

http://www.cambridgeincolour.com/tutorials/image-interpolation.htm
Nearest Neighboring Interpolation

**BILINEAR INTERPOLATION**

Bilinear interpolation considers the closest 2x2 neighborhood of known pixel values surrounding the unknown pixel. It then takes a weighted average of these 4 pixels to arrive at its final interpolated value. This results in much smoother looking images than nearest neighbor.

The diagram to the left is for a case when all known pixel distances are equal, so the interpolated value is simply their sum divided by four.

**BICUBIC INTERPOLATION**

Bicubic goes one step beyond bilinear by considering the closest 4x4 neighborhood of known pixels — for a total of 16 pixels. Since these are at various distances from the unknown pixel, closer pixels are given a higher weighting in the calculation. Bicubic produces noticeably sharper images than the previous two methods, and is perhaps the ideal combination of processing time and output quality. For this reason it is a standard in many image editing programs (including Adobe Photoshop), printer drivers and in-camera interpolation.

**HIGHER ORDER INTERPOLATION: SPLINE & SINC**

There are many other interpolators which take more surrounding pixels into consideration, and are thus also much more computationally intensive. These algorithms include spline and sinc, and retain the most image information after an interpolation. They are therefore extremely useful when the image requires multiple rotations / distortions in separate steps. However, for single-step enlargements or rotations, these higher-order algorithms provide diminishing visual improvement as processing time is increased.

http://www.cambridgeincolour.com/tutorials/image-interpolation.htm
In many fluid mechanical applications the velocity information by itself is of secondary interest in the physical description, which is principally due to the lack of simultaneous pressure and density field measurements. In general the pressure, density and velocity fields are required to completely recover all terms in the Navier–Stokes equation:

$$\rho \frac{DU}{Dt} = -\nabla p + \mu \nabla^2 U + F$$

Efforts to obtain some of these field quantities in addition to the velocity field is subject of current research. Clearly, the task of obtaining all of these field quantities simultaneously is a remaining challenge. By itself, the planar velocity field obtained by PIV can already be used to estimate other fluid mechanically relevant quantities by means of differentiation or integration which will be outlined in the following.
Estimation of Differential Quantities

Standard PIV data provide only the two components (U, V) of the three-dimensional vector field while more advanced PIV methods like stereoscopic PIV provide three-component velocity data. In order to see which differential terms actually can be calculated, the full velocity gradient tensor or deformation tensor, \( \frac{d\mathbf{U}}{d\mathbf{X}} \), will be given first:

\[
\frac{d\mathbf{U}}{d\mathbf{X}} = \begin{bmatrix}
\frac{\partial U}{\partial X} & \frac{\partial V}{\partial X} & \frac{\partial W}{\partial X} \\
\frac{\partial U}{\partial Y} & \frac{\partial V}{\partial Y} & \frac{\partial W}{\partial Y} \\
\frac{\partial U}{\partial Z} & \frac{\partial V}{\partial Z} & \frac{\partial W}{\partial Z}
\end{bmatrix}
\]

Symmetric term

\[
= \begin{bmatrix}
\frac{\partial U}{\partial X} & \frac{1}{2} \left( \frac{\partial V}{\partial X} + \frac{\partial U}{\partial Y} \right) & \frac{1}{2} \left( \frac{\partial W}{\partial X} + \frac{\partial U}{\partial Z} \right) \\
\frac{1}{2} \left( \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right) & \frac{\partial V}{\partial Y} & \frac{1}{2} \left( \frac{\partial W}{\partial Y} + \frac{\partial V}{\partial Z} \right) \\
\frac{1}{2} \left( \frac{\partial U}{\partial Z} + \frac{\partial W}{\partial X} \right) & \frac{1}{2} \left( \frac{\partial V}{\partial Z} + \frac{\partial W}{\partial Y} \right) & \frac{\partial W}{\partial Z}
\end{bmatrix}
\]

Asymmetric term

\[
+ \begin{bmatrix}
0 & \frac{1}{2} \left( \frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y} \right) & \frac{1}{2} \left( \frac{\partial W}{\partial X} - \frac{\partial U}{\partial Z} \right) \\
\frac{1}{2} \left( \frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X} \right) & 0 & \frac{1}{2} \left( \frac{\partial W}{\partial Y} - \frac{\partial V}{\partial Z} \right) \\
\frac{1}{2} \left( \frac{\partial U}{\partial Z} - \frac{\partial W}{\partial X} \right) & \frac{1}{2} \left( \frac{\partial V}{\partial Z} - \frac{\partial W}{\partial Y} \right) & 0
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\varepsilon_{XX} & \frac{1}{2} \varepsilon_{XY} & \frac{1}{2} \varepsilon_{XZ} \\
\frac{1}{2} \varepsilon_{YX} & \varepsilon_{YY} & \frac{1}{2} \varepsilon_{YZ} \\
\frac{1}{2} \varepsilon_{ZX} & \frac{1}{2} \varepsilon_{ZY} & \varepsilon_{ZZ}
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
0 & \frac{1}{2} \omega_Z & -\frac{1}{2} \omega_X \\
-\frac{1}{2} \omega_Z & 0 & \frac{1}{2} \omega_Y \\
\frac{1}{2} \omega_X & \frac{1}{2} \omega_Y & 0
\end{bmatrix}
\]
Estimation of Differential Quantities

\[ \omega_Z = \frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y} \]

\[ \epsilon_{XY} = \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \]

\[ \eta = \epsilon_{XX} + \epsilon_{YY} = \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} \]

Assuming incompressibility, that is, \( \nabla \cdot \mathbf{U} = 0 \), the sum of the in-plane extensional strains can be used to estimate the out-of-plane strain \( \epsilon_{ZZ} \):

\[ \epsilon_{ZZ} = \frac{\partial W}{\partial Z} = -\frac{\partial U}{\partial X} - \frac{\partial V}{\partial Y} \]

The recently introduced multiplane stereo PIV technique can be used to estimate the full vorticity vector.
# Standard Differential Scheme

<table>
<thead>
<tr>
<th>Operator</th>
<th>Implementation</th>
<th>Accuracy</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward difference</td>
<td>$\left( \frac{df}{dx} \right)<em>{i+1/2} \approx \frac{f</em>{i+1} - f_i}{\Delta X}$</td>
<td>O($\Delta X$)</td>
<td>$\approx 1.41 \frac{\varepsilon U}{\Delta X}$</td>
</tr>
<tr>
<td>Backward difference</td>
<td>$\left( \frac{df}{dx} \right)<em>{i-1/2} \approx \frac{f_i - f</em>{i-1}}{\Delta X}$</td>
<td>O($\Delta X$)</td>
<td>$\approx 1.41 \frac{\varepsilon U}{\Delta X}$</td>
</tr>
<tr>
<td>Central difference</td>
<td>$\left( \frac{df}{dx} \right)<em>i \approx \frac{f</em>{i+1} - f_{i-1}}{2\Delta X}$</td>
<td>O($\Delta X^2$)</td>
<td>$\approx 0.7 \frac{\varepsilon U}{\Delta X}$</td>
</tr>
<tr>
<td>Richardson Extrapol.</td>
<td>$\left( \frac{df}{dx} \right)<em>i \approx \frac{f</em>{i-2} - 8f_{i-1} + 8f_{i+1} - f_{i+2}}{12\Delta X}$</td>
<td>O($\Delta X^3$)</td>
<td>$\approx 0.95 \frac{\varepsilon U}{\Delta X}$</td>
</tr>
<tr>
<td>Least squares</td>
<td>$\left( \frac{df}{dx} \right)<em>i \approx \frac{2f</em>{i+2} + f_{i+1} - f_{i-1} - 2f_{i-2}}{10\Delta X}$</td>
<td>O($\Delta X^2$)</td>
<td>$\approx 1.0 \frac{\varepsilon U}{\Delta X}$</td>
</tr>
</tbody>
</table>

The “accuracy” in this table reflects the truncation error associated with derivation of each operator by means of **Taylor series expansion**.
Taylor Series Expansion

The Taylor series of a real or complex function \( f(x+h) \) that is infinitely differentiable in a neighborhood of \( x \)

\[
f(x + h) = f(x) + f'(x)h + \frac{f''(x)}{2!}h^2 + \frac{f'''(x)}{3!}h^3 + \ldots
\]

\[
f(x - h) = f(x) - f'(x)h + \frac{f''(x)}{2!}h^2 - \frac{f'''(x)}{3!}h^3 + \ldots
\]

\[
f(x) = f(a) + \frac{f'(a)}{1!}(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \frac{f'''(a)}{3!}(x - a)^3 + \ldots.
\]

\[
\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n
\]
function \to polynomial expression

\[
\sin (x) \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}.
\]

\[
(1-x)^{-1} \approx 1 + x + x^2 + x^3 + \cdots
\]
Forward Difference

\[ f'(x) = \frac{f(x + h) - f(x)}{h} - \frac{f''(x)}{2!}h - \ldots \]

\[ \approx \frac{f(x + h) - f(x)}{h} \]

Backward Difference

\[ f'(x) = \frac{f(x) - f(x - h)}{h} + \frac{f''(x)}{2!}h + \ldots \]

\[ \approx \frac{f(x) - f(x - h)}{h} \]

Central Difference

\[ f'(x) = \frac{f(x + h) - f(x - h)}{2h} - \frac{f'''(x)}{3!}h^2 + \ldots \]

\[ \approx \frac{f(x + h) - f(x - h)}{2h} \]

Central Difference

\[ f''(x) = \frac{f(x + h) - 2f(x) + f(x - h)}{h^2} - \frac{f^{(4)}(x)}{12}h^2 + \ldots \]

\[ \approx \frac{f(x + h) - 2f(x) + f(x - h)}{h^2} \]
Vorticity field estimates. The interrogation window overlap is 50%
Vorticity field estimates. The interrogation window overlap is 75%
Implementation of Differential Quantities with PIV Data

a) Vorticity

\[ \frac{dV}{dx} > 0 \]
\[ \frac{dU}{dy} < 0 \]

b) Shear strain

\[ \frac{dV}{dx} > 0 \]
\[ \frac{dU}{dy} > 0 \]

c) Normal strain

\[ \frac{dU}{dx} > 0 \]
\[ \frac{dV}{dy} > 0 \]

\[ (\vec{U} \cdot \delta s) \]

\[ (\vec{U} \cdot \hat{n}) \delta s \]
Finite Difference

\[
(\varepsilon_{xy})_{i,j} = \left( \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right)_{i,j} \approx \frac{-U_{i-1,j-1} + 2U_{i,j-1} + U_{i+1,j-1}}{8\Delta Y} + \frac{U_{i+1,j+1} + 2U_{i,j+1} + U_{i-1,j+1}}{8\Delta Y} - \frac{V_{i-1,j+1} + 2V_{i-1,j} + V_{i-1,j-1}}{8\Delta X} + \frac{V_{i+1,j-1} + 2V_{i+1,j} + V_{i+1,j+1}}{8\Delta X}
\]

\[
(\omega_z)_{i,j} \approx \frac{\Gamma_{i,j}}{4\Delta X \Delta Y}
\]

\[
\Gamma_{i,j} = \frac{1}{2} \Delta X (U_{i-1,j-1} + 2U_{i,j-1} + U_{i+1,j-1}) + \frac{1}{2} \Delta Y (V_{i+1,j-1} + 2V_{i+1,j} + V_{i+1,j+1}) - \frac{1}{2} \Delta X (U_{i+1,j+1} + 2U_{i,j+1} + U_{i-1,j+1}) - \frac{1}{2} \Delta Y (V_{i-1,j+1} + 2V_{i-1,j} + V_{i-1,j-1})
\]
Interrogation Window Size

Actual velocity profile

Measured velocity profile (Large interrogation window)

Measured velocity profile (Small interrogation window)

Actual vorticity profile

Estimated vorticity profile

Estimated vorticity profile
Estimation of Integral Quantities

Path Integrals – Mass Flow:

In some applications the rate mass or volume across a control surface, $CS$, is of interest and is expressed as a surface integral:

$$\dot{M} = \frac{dm}{dt} = \iint_{CS} \rho (U \cdot \hat{n}) \, dS$$

For two-dimensional data constrained to a $x$-$y$ plane, the surface reduces to a path integral (Green Theorem):

$$\dot{M}_{XY} = \frac{dm_{XY}}{dt} = \oint_{C} \rho (U \, dY - V \, dX)$$
Area Integrals

The following integration schemes are based on the assumption that the integrand, that is, the flow field, is two-dimensional as well as incompressible. Velocity field, $\mathbf{U} = (U(X, Y), V(X, Y))$, to the stream function, $\Psi$, and potential function, $\Phi$:

$$U = \frac{\partial \Psi}{\partial Y} = \frac{\partial \Phi}{\partial X}$$

$$V = -\frac{\partial \Psi}{\partial X} = \frac{\partial \Phi}{\partial Y}$$

$$\Psi = \int_Y U \, dY - \int_X V \, dX$$

$$\Phi = \int_X U \, dX + \int_Y V \, dY$$
Pressure from PIV Data

If the flow field under investigation is nearly two-dimensional, steady (i.e., \(dU/dt = 0\)) as well as incompressible (i.e., \(d\rho/dt = 0\)) the pressure field can be estimated through the numerical integration of the steady Navier–Stokes equations in two-dimensional form:

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{1}{\rho} \frac{\partial p}{\partial X} + \nu \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)
\]

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{1}{\rho} \frac{\partial p}{\partial Y} + \nu \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right)
\]

To obtain the pressure field, the pressure gradients \(\partial p/\partial X\) and \(\partial p/\partial Y\) are approximated using finite difference approximations of the velocity gradients.
Pressure from PIV Data

To start the integration a starting point, $P_0$, is chosen, preferably near the middle of the velocity field since errors in the individual velocity data are propagated through integration. In the first case, the integration proceeds in opposite horizontal directions away from the starting point, $P_0$, producing new values of the integral for each node on the horizontal. These new estimates are then used as initial values for the integration in opposite directions along the vertical columns, producing estimates of the integral throughout the domain. A second estimate of the integral can then be obtained by reversing the order of the integration scheme, that is, by starting the integration off in opposite directions along the vertical line containing the starting point. The two results are then arithmetically averaged together.
Pressure and Forces from PIV Data

The availability of time-resolved data allows the proper treatment of the acceleration term in equation (1) as demonstrated by Liu & Katz on a 2D cavity turbulent flow field using a four frame PIV system.

\[
\begin{align*}
-\nabla p &= \rho \frac{dU}{dt} + \rho (U \cdot \nabla) U - \mu \nabla^2 U \\
- \frac{\partial p}{\partial x_i} &= \rho u_j \frac{\partial u_i}{\partial x_j} + \rho \frac{\partial \overline{u'_i u'_j}}{\partial x_j} - \mu \frac{\partial^2 u_i}{\partial x_j \partial x_j}
\end{align*}
\]
Vortex Detection

Though there seems to be a common understanding about what a vortex looks like, it is mostly defined by empirical arguments or may be very subjective. In general, vortices are created due to conservation of angular momentum and not necessarily assume an easily detectible circular shape, especially if several vortices interact with each other. A vortex may be characterized by its location, circulation, core radius, drift velocity, peak vorticity, maximum circumferential velocity, for instance.

\[
g = \frac{dU}{dX} = \begin{bmatrix} \frac{\partial U}{\partial X} & \frac{\partial V}{\partial X} \\
\frac{\partial U}{\partial Y} & \frac{\partial V}{\partial Y} \end{bmatrix}
\]

\[
det(g) = (dU/dX)(dV/dY) - (dV/dX)(dU/dY)
\]
3D PIV Measurements

Three Dimension Three Component (3D3C) Measurements

- Holographic PIV
- Defocusing PIV

Two Dimension Three Component (2D3C) Measurements

- Stereoscopic PIV
- Dual Plane PIV
Stereoscopic PIV
Velocity Field Reconstruction

The angle $\alpha$ in the $XZ$ plane

$\beta$ defines the angle within the $YZ$ plane.

\[ x'_i - x_i = -M \left( D_X + D_Z \frac{x'_i}{z_0} \right) \]

\[ y'_i - y_i = -M \left( D_Y + D_Z \frac{y'_i}{z_0} \right) \]

\[ \tan \alpha = \frac{x'_i}{z_0} \]

\[ \tan \beta = \frac{y'_i}{z_0} \]

\[ U_1 = -\frac{x'_i - x_i}{M \Delta t} \]

\[ V_1 = -\frac{y'_i - y_i}{M \Delta t} \]
Velocity Field Reconstruction

\[ U = \frac{U_1 \tan \alpha_2 + U_2 \tan \alpha_1}{\tan \alpha_1 + \tan \alpha_2} \]  
\[ V = \frac{V_1 \tan \beta_2 + V_2 \tan \beta_1}{\tan \beta_1 + \tan \beta_2} \]  
\[ W = \frac{U_1 - U_2}{\tan \alpha_1 + \tan \alpha_2} \]  
\[ V = V_1 + V_2 + \frac{W}{2} \left( \tan \beta_1 - \tan \beta_2 \right) \]  
\[ V = \frac{V_1 + V_2}{2} + \frac{U_1 - U_2}{2} \left( \frac{\tan \beta_1 - \tan \beta_2}{\tan \alpha_1 + \tan \alpha_2} \right) \]

If the tangents \( \tan \beta_1 \) and \( \tan \beta_2 \) very small, component \( W \) can only be estimated with higher accuracy using equation (2) while \( V \) has to be rewritten using equation (2) which does not include \( \tan \beta_1 \) and \( \tan \beta_2 \) in the denominator.
An example

http://www.eng.uwo.ca/people/rgurka/xpiv.htm
Calibration
Dual Plane PIV

One camera with two light sheets.
Velocity Field Reconstruction

\[ E\{R_{II'}(s = d)\} = C_R \ R_\tau(0) \ F_0(D_Z - Z' + Z) \ F_i(D_X, D_Y) \]

\[ E\{R_{I'I''}(s = d)\} = C_R \ R_\tau(0) \ F_0(D_Z - Z'' + Z') \ F_i(D_X, D_Y) \]

\[ F_0(D_Z) = \frac{\int I_0(Z)I_0(Z + D_Z) \ dZ}{\int I_0^2(Z) \ dZ} \]

\[ \frac{R_{II'}}{R_{I'I''}} = \frac{F_0(D_Z - Z' + Z)}{F_0(D_Z - Z'' + Z')} \]
Velocity Field Reconstruction

\[ I_0(Z) = \begin{cases} 
I_Z & \text{if } |Z| \leq \Delta Z_0/2 \\
0 & \text{elsewhere}
\end{cases} \]

and

\[ F_0(Z) = \begin{cases} 
1 - |Z|/\Delta Z_0 & \text{if } |Z| \leq \Delta Z_0 \\
0 & \text{elsewhere}
\end{cases} \]

\[ W = \frac{\Delta Z}{\Delta t} \left\{ \begin{array}{c}
\frac{R_{IV'} - O_Z R_{IV}}{R_{II'} - R_{IV''}} \\
\frac{R_{IV'} - O_Z R_{II}}{R_{II'} + R_{IV''}} \\
\frac{R_{IV''} + (2 - O_Z) R_{IV'}}{R_{II'} - R_{IV''}}
\end{array} \right\} 
\begin{array}{c}
\text{for } -\Delta Z \cdot O_Z \leq D_Z \leq 0 \\
\text{for } 0 \leq D_Z \leq Z'' - Z' \\
\text{for } Z'' - Z' \leq D_Z \leq \Delta Z
\end{array} \]

\[ W = \frac{\Delta Z}{\Delta t} \frac{R_{IV'} - R_{IV}}{R_{II'} + R_{IV''}} O_Z \]
An Example

http://www.public.iastate.edu/~huhui/research/dual-plane-spiv/dp-spiv.html