Difficulties involving dynamic polarization-based impairment measurements using Jones matrices

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The concurrent realization of multiple polarization-based link impairments such as polarization-mode dispersion, polarization-dependent loss, and differential-attenuation slope creates a nontrivial measurement and analysis problem. The difficulties concerning the robustness of raw data, of minimizing drift artifacts experienced during multiple wavelength measurements, and the analysis methods that lead to physical significant interpretations are addressed. Measurements of an in-service wavelength-division-multiplexed metro-area network are presented that explicitly illustrate the limitations when using industry-standard commercial test equipment. © 2004 Optical Society of America

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1. INTRODUCTION

Polarization-based impairments can be generalized into those signal-degrading phenomena that interact differently between polarization states. Polarization-based link impairments in telecommunication systems have been classified into several categories: polarization-mode dispersion, polarization-dependent loss, and polarization-dependent gain. These are vector quantities that can degrade the optical signal of a fiber-optic communication link. In the laboratory, these impairments are typically considered as slowly varying and independent sources of signal degradation. However, real-world situations exist where more than one impairment may be present, and the system may also be changing in a dynamic fashion. This paper addresses three difficulties associated with measurements of polarization-based impairments that make link assessments more challenging than simply applying techniques that were developed in the laboratory to real-world systems.

The first difficulty associated with any measurement is to determine the robustness of the data. The metric presented here uses the correlation between the polarization-dependent loss (PDL) and the time-dependent rotation rate of the output state of polarization (SOP) to test the effects of a rapidly varying output SOP on the accuracy of a single Jones measurement. This correlation is selected because PDL and SOP can be determined from a single Jones matrix and are expected to be uncorrelated. We define the error in a single Jones measurement that is caused by a rapidly varying system to be the dynamic artifact.

The second difficulty associated with link assessments of a real-world system is that concerning the nonstationary behavior of the system. The temporal evolution of polarization-mode dispersion (PMD) in an installed submarine fiber has been documented and has been attributed to polarization-mode coupling that causes the fiber PMD to vary with temperature and environmental conditions. Other field measurements of PMD also show large variations of PMD as a function of time and wavelength. Many of these studies are completed with large-wavelength scans that take hours to repeat. The extent to which the drift in these systems that occurs on the time scale comparable to the time needed to make several measurements that are needed for analysis is not clear. Here, a data interpolation procedure is presented that reduces the drift artifact, which is our definition of the error resulting from confusion between time-dependent and optical-frequency-dependent phenomena that occurs when examining an evolving system.

The third difficulty impeding the accurate assessment of an in-service transmission link is the relevance of the numerical analysis. It has been explicitly shown in simulation that polarization-based impairments can interact and produce a situation that is worse than the simple superposition of independent impairments. Accordingly, a realistic link assessment of polarization-
based impairments in a real-world setting must address the presence of every possible impairment. We review the conventional application of the Jones-matrix eigenanalysis and show its limitations in the presence of other polarization-based impairments. We also show that a specific form of the polar decomposition can be used to obtain physically significant results, and we present a means to apply the Pauli-matrix decomposition to measured data such that it is another alternative that yields physically significant results.

The techniques developed in this paper are applied to measurements acquired on the ATDnet, which is an in-service wavelength-division-multiplexed (WDM) metro network. In contrast to other published measurements of installed fiber links, the measurements reported here are acquired on a single WDM channel and are measured at one-minute intervals instead of several-hour intervals. This sampling rate provides a higher temporal resolution for interrogating the dynamic evolution of this system.

While conducting a link assessment, these difficulties need to be considered in the order described above. However, for readability purposes, they are presented in reverse order so that the motivation and significance of each physical property is clearly presented. The experimental results obtained on two separate links of the metro-area network under test clearly illustrate the need to consider the measurement difficulties described in this paper for every link assessment.

2. JONES-MATRIX ANALYSIS

There are numerous techniques that have been shown to make accurate measurements on isolated impairments; this paper focuses on Jones-matrix-based measurement techniques because they allow vector information to be obtained from the measurements, and, as shown here, Jones matrices provide sufficient information to allow the determination of all impairments from a few measurements using one instrument.

A Jones matrix describes the polarization-dependent effect of light traveling through a transmission system. For a general complicated system, the Jones matrix does not contain sufficient information to uniquely decompose the transmission system into its constitutive components (i.e., fiber, amplifiers, and WDM couplers). Therefore it is important to remember that any analysis of a fiber link is a measure of its effective properties and may not necessarily lead one to easily decompose the measurement results to determine the source or sources of a specific result.

A. Presence of Polarization-Mode Dispersion

Polarization-mode dispersion (PMD) is the dispersion between polarization states that results from a nonsymmetrical mode shape. PMD is seen to be the major system impairment that will limit the transmission lengths of optical-fiber spans in current- and next-generation communication systems, and it will also determine the feasibility of upgrading components of optical networks around legacy optical fibers. The physical sources of polarization-mode dispersion are those that cause an anisotropic perturbation away from the ideal cylindrically symmetric optical core. The perturbations of the core may be caused by internal stresses or irregularities caused during the manufacturing process or the result of a nonuniform stress applied to the fiber. They may also depend on the temperature of the fiber or on disturbances from the environment. While the local birefringence of the optical fiber may be small, the large propagation length of a fiber span may be large enough that the differential group delay (DGD) between orthogonal polarization states can be a sizeable fraction of a bit period. Clearly, as bit rates of the transmission system increase, the tolerance of these perturbations (i.e., DGD) decreases, resulting in a smaller margin for error. Therefore it is important to study polarization-mode dispersion so that it can be actively mitigated, designed around using modulation formats with reduced sensitivity, or minimized by installing new low-PMD fibers that are routed in such a fashion that they are isolated from environmental influences. There have been many approaches to measuring PMD. Techniques can be classified into frequency-domain measurements such as the Poincare-sphere method, which examines the motion of the output SOP as a function of wavelength, the Jones-matrix eigenanalysis (JME), which analyzes the changes in measurements of the Jones matrix of the transmission system as a function of wavelength, and time-domain techniques such as the modulation phase-shift method, which measures the phase variation between the slow and fast transmission axes. This paper does not contain a complete description of these techniques, and the reader should refer to other sources for an unabridged list.

Originally developed by Poole, the Jones-matrix eigenanalysis (JME) determines the pair of output SOPs that are constant in frequency. Because the Jones-matrix eigenanalysis is completed on a system that does not contain PDL, the Jones matrix, $J = U$, is unitary, which causes the JME operator, $JU U^{-1}$, to be Hermitian, which can be shown using the derivative of the unitary condition. This eigenvalue problem, $JU U^{-1} \{p\} = 1/2\tau \{p\}$, where $\tau$ is the DGD and the eigenvectors, $\{p\}$, are the principle states of polarization (PSP). The physical significance of the principal states is that they represent orthogonal output states that have a frequency-independent polarization state, and the difference between the eigenvalues is the differential group delay, or the propagation delay between the two principal states.

This interpretation is consistent with the PMD dynamical equation in Stokes space,

$$\frac{d\mathbf{S}}{d\omega} = \Omega \times \mathbf{S},$$

where $\mathbf{S}$ is the SOP, $\omega$ is the optical frequency, and the PMD vector, $\Omega$, has a magnitude equal to the DGD and a direction that is defined by the larger principle state. The nontrivial solutions for stationary SOP using Eq. (1) are for the cases where $\mathbf{S}$ and $\Omega$ are either parallel or antiparallel. Stokes space is preferred over Jones-matrix space for visualizing the SOP behavior because it is a three-dimensional real vector space. If one solves the PMD dynamical equation in Stokes space, or converts the
The all-states method, Mueller stalled fiber systems. A number of techniques have been analyzed in addition to the DGD and the PDL for the in-
as the differential-attenuative slope (DAS) and should be dependence of polarization-dependent loss is quantified the DGD, the PDL can become frequency dependent and
begin to act as a spectral filter. The linear wavelength dependence of polarization-dependent loss is quantified as the differential-attenuative slope (DAS) and should be analyzed in addition to the DGD and the PDL for the in-
stalled fiber systems. A number of techniques have been demonstrated to quantify PDL. These methods, such as the all-states method, Mueller–Stokes method, and Jones-matrix method, are different ways to account for all possible input states of polarization and to measure variations in output power.

Analysis of complex transmission links that exhibit multiple polarization-based impairments is not as straightforward as applying the Jones-matrix eigenanalysis. When a small amount of PDL is present, the Jones matrix, \( J \), no longer satisfies the unitary condition, and therefore the PMD operator is not Hermitian. The eigenvectors and eigenvalues of the operator, \( jJ, oJ^{-1} \), are in general complex and not orthogonal. These states still describe the output SOPs that are constant in frequency, but no further physical significance can be assigned. That is, the result of the Jones-matrix eigenanalysis when PDL is present is the nontrivial solution of a stationary SOP (\( \partial S/\partial \omega = 0 \)) in the dynamical PMD equation presented by Li and Yariv,

\[
\frac{\partial S}{\partial \omega} = \Omega \times S - (A \times S) \times S, \tag{2}
\]

which occurs when the rotation from birefringence around the PMD vector \( \Omega \) is canceled by the equal and opposite rotation associated with the DAS vector \( \Lambda \).

A simple demonstration of the error associated with analysis using JME when DAS is present for the Jones matrix, \( J \), can be demonstrated using a first-order PMD section defined by the diagonal matrix, \( V \), and a first-order DAS section described by the diagonal matrix, \( L \), which is rotated with respect to the PMD section as follows:

\[
J = VRLR^{-1}, \tag{3}
\]

where

\[
V = \begin{bmatrix}
\exp(-i \omega_0 \tau_0/2) & 0 \\
0 & \exp(i \omega_0 \tau_0/2)
\end{bmatrix},
\]

\[
R = \begin{bmatrix}
\cos(\phi) & -\sin(\phi) \\
\sin(\phi) & \cos(\phi)
\end{bmatrix},
\]

\[
L = \begin{bmatrix}
\exp(-\chi_0 \omega_0/2) & 0 \\
0 & \exp(\chi_0 \omega_0/2)
\end{bmatrix}. \tag{4}
\]

If \( \phi = 45° \), and the Jones matrix of Eq. (3) is directly analyzed using the JME technique, the eigenvalues of \( \lambda = \pm (\chi_0^2 - \tau_0^2) / 4 \) are found. When the magnitude of the DAS and the DGD vectors are equal, the JME results yield zero polarization-based impairments. In contrast to this, consider a linearly polarized Gaussian pulse,

\[
A(\omega) = \frac{\cos(\theta)}{\sin(\theta)} \left[ \exp\left[ -\frac{(\omega - \omega_0)^2}{2 \tau^2} \right] \right], \tag{5}
\]

where \( \omega \) is the offset frequency around a center frequency (\( \omega_0 \)) that is propagated through \( J \) and examined in the time domain using the Fourier transform \( O(t) = F^{-1}[JA] \). The total output intensity in the time domain is shown in Fig. 1 for \( \theta = 22.5° \) and \( \omega_p = 50 \text{ GHz} \), when \( \tau_0 = \chi_0 = 30 \text{ ps} \), when \( \chi_0 = 0 \) and \( \tau = 30 \text{ ps} \), and when \( \tau_0 = 0 \text{ ps} \) and \( \chi_0 = 30 \text{ ps} \). Even though the JME yields zero impairments for the case when \( \tau_0 \) is equal to \( \chi_0 \), it is apparent from Fig. 1 that the pulses propagating through this simple system experience the same relative delays whether or not there is any DAS. The realization of the erroneous assessment resulting from the neglect of DAS is important. Furthermore, it is apparent that the rotations caused by DAS will also cause erroneous assessment of the DGD for all other measurement techniques that use Eq. (2) with the assumption that only birefringence is present (i.e., \( \Lambda = 0 \)).

Huttner\(^9\) and Karlsson\(^20\) addressed this problem by applying a polar decomposition to the measured Jones matrix in order to separate the birefringence rotations that are caused by frequency-dependent PDL. A polar decomposition\(^21,22\) can be used to separate any matrix into a Hermitian modulus (\( H \) or \( L \)) and a unitary matrix (\( U \) or \( V \)) in one of two orderings:

\[
J = HU, \quad \text{or} \quad J = VL. \tag{6}
\]

![Fig. 1. Comparison of pulse distortion when passing through a PMD and DAS components that are oriented at 45° with respect to each other. The Gaussian pulse is introduced at the median angle between the components. The output when \( \tau_0 = 30 \text{ ps} \) and \( \chi_0 = 30 \text{ ps} \) (solid), when \( \tau_0 = 30 \text{ ps} \) and \( \chi_0 = 0 \text{ ps} \) (dashed), and when \( \tau_0 = 0 \text{ ps} \) and \( \chi_0 = 30 \text{ ps} \) (dotted). Even though the JME yields zero impairments for the solid curve, it is apparent that the pulses propagating through this simple system are degraded more than the case of only DGD.](image-url)
While the two decompositions describe the same Jones matrix, the left and right Hermitian moduli (H and L, respectively) are not equivalent; the unitary matrices (U and V) are not equivalent and are only unique if the Jones matrix is nonsingular. Nevertheless, this decomposition is useful because it allows an arbitrary Jones matrix to be decomposed into effective loss and dispersion matrices, which can be independently analyzed. Consider the Jones-matrix eigenanalysis written in terms of the right-hand-side (RHS) decomposition:

$$jJ_{\omega}^{-1} = jV_{\omega}V^{-1} + jVL_{\omega}L^{-1}V^{-1}. \quad (7)$$

Assuming that the Hermitian matrix can be diagonalized to $A(\omega)$ with a suitable rotation $R(\omega)$ by $V = RAR^{-1}$, then

$$jL_{\omega}L^{-1} = jR_{\omega}R^{-1} + jRA_{\omega}A^{-1}R^{-1} + jRAR_{\omega}^{-1}RA_{\omega}^{-1}R^{-1}. \quad (8)$$

Since $R$ is unitary, the first RHS term in Eq. (8) is Hermitian. Because they are diagonal matrices, the product $A_{\omega}A^{-1}$ commutes and is therefore a Hermitian product. The unitary rotation matrices surrounding this product do not change the eigenvalues, but $j$ makes these values complex (but maintains orthogonality between the eigenvectors), and thus the second RHS term is skew Hermitian. The third RHS term is the product of two Hermitian matrices, and in general cannot be classified as Hermitian or skew Hermitian. Therefore only in the case of frequency-independent eigenvectors ($R_{\omega} = 0$) is the operator $jL_{\omega}L^{-1}$ skew Hermitian.

In the case of frequency-independent eigenvectors of the DAS, the polar decomposition has allowed for independent analysis of the DGD by analyzing the Hermitian operator defined with the derivative of the unitary component, and the DAS using the skew-Hermitian operator defined with the derivative of the Hermitian component. The Jones-matrix eigenanalysis problem of PMD and PDL written by Li and Yariv\(^{19}\) can be separated into independent equations:

$$jV_{\omega}V^{-1} = \frac{1}{2} \Omega \cdot \sigma, \quad (9)$$

$$jVL_{\omega}L^{-1}V^{-1} = \frac{j}{2} \Lambda \cdot \sigma. \quad (10)$$

where $\Omega$ is the PMD vector, $\Lambda$ is the DAS vector, and $\sigma$ is the right-handed Pauli spin vector of Gordon and Kogelnik,\(^{15}\) which is defined as

$$\sigma = (\sigma_1, \sigma_2, \sigma_3). \quad (11)$$

where

$$\sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}. \quad (12)$$

When both the DAS and DGD vectors both have frequency-dependent rotations, the separation of $J$ using the polar decomposition breaks down because the second term in Eq. (7) describes a mixed state with complex eigenvalues and nonorthogonal eigenvectors. The eigenvalue problem now has the same issues that affected the straightforward application of the JME, which results in a loss of physical meaning. It is therefore important that the frequency step size is small enough that the impairments are accurately described by a first-order approximation.

Looking at the left-sided decomposition,

$$jJ_{\omega}^{-1} = jH_{\omega}H^{-1} + jHU_{\omega}U^{-1}H^{-1} \quad (13)$$

yields a different result. The second RHS term of Eq. (13), which is the product of three Hermitian matrices ($H$, $U_{\omega}U^{-1}$, and $H^{-1}$), cannot generally be classified as a Hermitian, or skew-Hermitian operator. While this is a mathematically correct decomposition, the second term is a mixed state that has complex eigenvalues and eigenvectors that are not necessarily orthogonal. A simple decomposition similar to Eq. (9) cannot be written that allows for any additional physical insight. Therefore only the right-sided polar decomposition should be used for the analysis of Jones-matrix data.

An alternative method that directly yields results in Stokes space and does not require the solution to an eigenvalue problem is the Pauli-matrix equivalence technique. The Pauli-matrix equivalence technique uses symmetry to determine the vector components of the PMD and DAS vectors and has been used in analytical work\(^{15}\) and for the analysis of simulations,\(^{10}\) but has not previously been used for experimental analysis because the Pauli spin matrices form a traceless basis that provides a means of decomposing a traceless matrix from a $2 \times 2$ complex vector space (Jones space) into a vector contained in a real $3 \times 3$ vector space (Stokes space). As described in the original paper by Jones,\(^{23}\) the Jones matrix, $J_{\omega}$, for an arbitrarily measured Jones matrix is

$$J_{\omega} = C(\omega) \begin{bmatrix} a(\omega) + ib(\omega) & c(\omega) + id(\omega) \\ e(\omega) + i\phi(\omega) & 1 \end{bmatrix}, \quad (14)$$

which is not traceless and does not yield a traceless PMD operator, i.e., $\text{Tr}(J_{m}J_{m}^{-1}) \neq 0$, and therefore cannot be directly expressed using only the Pauli spin vector. If the identity matrix, $I$, is used as $\sigma_0 = I$ to augment the Pauli spin matrices, the Jones matrix can be written as

$$J(\omega) = \exp \left\{ \sum_{k=0}^{3} [a_k(\omega) + i\phi_k(\omega)] \sigma_k \right\}. \quad (15)$$

and the eigenvalues, $\xi_1$ and $\xi_2$, of the Jones matrix can be written in a similar form,

$$\xi_1(\omega), \xi_2(\omega) = \exp \left\{ \frac{1}{2} \left[ a_0(\omega) + i\phi_0(\omega) \right] \right\} \pm \sqrt{\sum_{k=1}^{3} [a_k(\omega) + i\phi_k(\omega)]^2}. \quad (16)$$

While the term containing $\sigma_0 = I$ is needed to describe a general Jones matrix, its inclusion eliminates the decomposition into a purely traceless basis. Since $\sigma_0$ and $\phi_0$ contribute symmetrically to the Jones matrix, they can be pulled outside of the matrix through the appropriate pre-
multiplier \(a(\omega)\), which is determined from the eigenvalues of the Jones matrix using
\[
J(\omega) = a(\omega)J_m(\omega)
\]
\[
= \exp \left( \frac{-\ln[\xi_1(\omega)] + \ln[\xi_2(\omega)]}{2} \right) J_m(\omega). \tag{17}
\]
The result of this operation produces a Jones matrix, \(J(\omega)\), that is not traceless itself, but will yield a traceless PMD operator \([\text{Tr}(J \omega J^{-1}) = 0]\), and a multiplicative modification to the common phase multiplier, \(C(\omega) \rightarrow C(\omega)a(\omega)\). The modification to the global premultiplier can be readily discarded because it is not experimentally determined. In most applications, these shortcomings are acceptable uncertainties; here, they provide a suitable multiplier that can be used to force \(J_m\) into a form such that the Pauli-matrix decomposition can be applied without sacrificing accuracy.

Now that the Jones matrix is in the appropriate form, the PMD vector, \(\Omega\), and the DAS vector, \(\Lambda\), can be determined in Stokes space using the PMD operator:
\[
J_\omega J^{-1} = \sum_{k=1}^{3} \frac{1}{2} \left( \frac{\partial}{\partial \omega} [a_k(\omega) + i \phi_k(\omega)] \sigma_k \right). \tag{18}
\]
The PMD vector is determined using
\[
\Omega_1 = -2\Im([J_\omega J^{-1}]_{11}),
\]
\[
\Omega_2 = -\Im([J_\omega J^{-1}]_{21} + [J_\omega J^{-1}]_{12}),
\]
\[
\Omega_3 = -\Im([J_\omega J^{-1}]_{21} - [J_\omega J^{-1}]_{12}). \tag{19}
\]
The vector components of the DAS vector are determined using
\[
\Lambda_1 = 2\Im([J_\omega J^{-1}]_{11}),
\]
\[
\Lambda_2 = \Re([J_\omega J^{-1}]_{21} + [J_\omega J^{-1}]_{12}),
\]
\[
\Lambda_3 = \Im([J_\omega J^{-1}]_{21} - [J_\omega J^{-1}]_{12}). \tag{20}
\]
while the DGD, \(\tau\), and the DAS, \(\chi\), are
\[
\tau = |\Omega|, \tag{21}
\]
\[
\chi = |\Lambda|. \tag{22}
\]
This method resolves the PMD and DGD vectors by equating the symmetric contribution of the matrix \(J_\omega J^{-1}\) with PMD and the antisymmetric contribution to DAS. While there is a direct connection between the DGD and DAS vectors when only first-order impairments are present, the connection between cause and effect is mixed when there is frequency dependence of the DGD and DAS vectors. In the most general perspective, the Pauli-matrix equivalence technique resolves the phenomena into an apparent DGD and an apparent DAS based on the nature of the rotation around the SOP.

C. Comparison of Techniques
To compare the different analysis techniques, consider the following simplified system with first-order PMD and DAS described in Eq. (3) for fixed values of \(\tau_0\) and \(\chi_0\). Figure 2 shows the results as a function of rotation (\(\phi\)) between the matrices \(L\) and \(V\). For comparison, the difference between the real part of the eigenvalues of the straightforward JME is interpreted as the DGD, while the difference between the imaginary components is considered the DAS. As described above, the straightforward application of the JME technique can produce values that are significantly less than the actual DGD and DAS because of the balance of the motion, and when these two are orthogonal on the Poincare sphere, the rotations on the SOP are opposite and the analyzed values are falsely zero. Alternatively, the Jones-matrix eigenanalysis following a polar decomposition and the Pauli-matrix equivalence analysis yield results that accurately describe the first-order system being investigated.

It should be noted that each analysis technique breaks down for a general higher-order description of the Jones matrix. It is therefore imperative that the wavelength steps be as small as possible, in order to minimize higher-order rotation of the SOP during measurements.

Looking at the polar decomposition JME and the Pauli-matrix equivalence analysis techniques, one concludes that the Pauli-matrix equivalence analysis should be more robust because it does not have the propensity of losing significance from nonorthogonal eigenvectors and complex eigenvalues. However, from the perspective of constructing a PMD and DAS compensator or emulator, the analysis of the polar decomposition JME technique provided a physical representation that can be constructed from a specific ordering of components.

3. MINIMIZING DRIFT ARTIFACTS BETWEEN MEASUREMENTS
While Jones-matrix techniques have the advantage of allowing for the various polarization effects to be separated, the current trade-off is acquisition time. Figure 3 shows a typical measurement scheme, where the Jones measurements are acquired for a series of wavelengths. These measurements require a tunable laser, a program-
able polarization controller, and a polarimeter. Specifics regarding the measurement are completely described elsewhere. The time elapsed between successive 

regarding the measurement are completely described 

able polarization controller, and a polarimeter. Specifics 

with the temporal drift in the system can be understood 

matrices themselves are accurate, the drift associated 

polarization-based impairments. Assuming the Jones 

tions of the Jones matrices affect the calculated 

acquisition, tuning rate of the laser source, and speed of 

Fig. 3. Measurement scheme typically followed while making 

wavelength-dependent measurements of Jones matrices. 

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which can be written as 

\[ \frac{dS}{\delta \omega} \delta \omega + \frac{\partial S}{\partial t} \delta t, \] 

which can be written as 

\[ dS = \left( \frac{\langle s | \sigma | s \rangle}{\langle s | s \rangle} \right) d\omega + \left( \frac{\langle s | \sigma | s \rangle}{\langle s | s \rangle} \right) dt. \] 

Using the identities 

\[ M = m_3 J + m \cdot \sigma, \] 

\[ \langle s | a \times \sigma | s \rangle = a \times \langle s | \sigma | s \rangle = a \times \sigma, \] 

\[ \langle s | a | s \rangle = a \times \langle s | s \rangle = a | s \rangle. \] 

The complete differential written as a dynamical equation is 

\[ dS = [\Omega \times S - (A \times S) \times S]d\omega \] 

\[ + [\Psi \times S - (Y \times S) \times S]dt. \]

The first term in Eq. (26) is the dynamical equation of 
PMD and DAS reported by Li and Yariv. The second 
term is the result of time dependent variation in the 
effective birefringence [V in Eq. (6)] and the effective loss 
[L in Eq. (6)]. The operators \( \Psi \times \) and \( (Y \times) \times \) describe 
the time dependence of the SOP. Since optical frequency 
and time are both real variables, insight can be gained 
through analogy of Eq. (7), where \( t \) replaces \( \omega \). That is, 
rotation around the SOP associated with \( \Psi \) are caused by 
changes in the birefringence, i.e., \( V \), and rotations of 
the loss \( L \). Rotation toward the SOP associated with \( Y \) are 
caused by changes in the magnitude and direction of \( L \).

Given that there are time-dependent variations in the 
Jones matrix under investigation, there are two possibilities 
that will be described here that can be used to minimize 
the errors associated with the temporal evolution of the 
polarization-based impairments using existing equipment 
and measurement practices. It is important to note 
that the measurement period effectively samples the real-world 
system; therefore the Nyquist limit of the maximum 
temporal variations that can be compensated are 
determined by the acquisition time of the measurement 
system.

The most simple approach to remove a sporadic tempo-
ral variation in the measured data is to monitor an esti-
mate of the error associated with temporal rotations 
compared with wavelength rotations. A simple error 
estimate can be made by purposely attributing the tem-
poral rotation to a wavelength rotation such as
tion matrices, while $V$ and $L$ are defined using Eq. (4). This model was selected because it closely resembles what might happen for a nonuniform fiber link, where one particular section may be coupled to the environment causing evolution in the SOP without dramatically modifying the DGD and DAS. The rotation matrices $P$ and $Q$ determine the orientations of the PMD and DAS vectors, while $R$ is used to simulate the system evolution resulting from only dynamic SOP rotation. To accomplish this, $R$ is modeled as $R(t_{n}) = \delta R + R(t_{n-1})$, where $\delta R$ is a small-perturbation rotation vector whose three components are determined from 1000 instances of uncorrelated normally distributed sets. Real systems are continuous and have a smoothly varying SOP, but since this model is discrete and simulates temporal evolution with instantaneous perturbations, the individual components of the Jones matrices are independently smoothed in the time dimension with a ten-point adjacent average filter such that the model is similar to a continuously varying system and to ensure that the autocorrelation width is larger than the extent of the difference scheme. This method satisfies the criteria of a randomly varying rotation that is checked through the SOP that has zero-mean normally distributed vector components, and the dot product of adjacent SOP vectors that are described by a Rayleigh distribution.

The error associated with analysis of the simulated dynamically evolving system is shown in Figs. 4 and 5. The error is defined as the mean absolute value of the difference between the analyzed DGD and DAS and that used to form $J$ as defined in the previous paragraph. Calculations for 100 different orientations of the DGD and DAS (i.e., 100 different rotation matrices $P$ and $Q$) without interpolation are shown in gray in Fig. 4 for $\tau = 1$ ps and $\chi = 1$ ps, and in Fig. 5 for $\tau = 10$ ps and $\chi = 1$ ps. The DGD and DAS that result from analysis of these matrices with isochronal analysis made possible by interpolating the Jones matrices are shown in black in the same figures. Two observations are worth noting. First, the sensitivity to temporal dynamics depends on the magnitude of the DGD and DAS but not necessarily on their relative orientations. Second, the use of interpolated data causes an order of magnitude reduction to the sensitivity to temporal rotations.

4. ROBUSTNESS OF DATA

In the previous section, it was assumed that the individual Jones matrices were accurately determined. This may be an accurate assumption if the temporal evolution of the system is slowly varying. However, it is quite possible that a dynamic system may have variations that affect the accuracy of an individual Jones matrix. In such a case, an examination of PDL statistics provides a practical means of determining the extent of dynamic artifacts of the measurement. Because PDL can be determined from a single Jones measurement (compared with temporal- or frequency-dependent rotations), it is physically observable and catches a snapshot of the transmission line under test as seen through the measurement apparatus. The PDL of the fiber can be determined directly from any Jones matrix using the components of the Pauli matrix decomposition of the transmittance as described in Ref. 22, which is included here for completeness:
PDL = 10 \log \left( \frac{\beta_0 + \sqrt{\beta_1^2 + \beta_2^2 + \beta_3^2}}{\beta_0 - \sqrt{\beta_1^2 + \beta_2^2 + \beta_3^2}} \right), \quad (29)

where

\begin{align*}
\beta_0 &= \frac{1}{2} (|J_{11}|^2 + |J_{12}|^2 + |J_{21}|^2 + |J_{22}|^2), \\
\beta_1 &= \frac{1}{2} (|J_{11}|^2 - |J_{12}|^2 + |J_{21}|^2 - |J_{22}|^2), \\
\beta_2 &= \frac{1}{2} (J_{11}^* J_{12} + J_{21}^* J_{22}), \\
\beta_3 &= \frac{1}{2} (J_{11}^* J_{12} + J_{21}^* J_{22}),
\end{align*}

(30)

and the PDL unit vector is defined as

\[ \hat{\beta} = \frac{(\beta_1, \beta_2, \beta_3)}{\sqrt{\beta_1^2 + \beta_2^2 + \beta_3^2}}. \quad (31) \]

Using industry-standard equipment, the measurement of a single Jones measurement is acquired over the course of approximately one second. This time is needed to launch three input polarization states and to measure the corresponding output states of polarization. If the Jones matrix of a transmission line changes over the course of this acquisition time, then the time-dependent rotations can cause the recorded Jones matrix to artificially represent a system with PDL. This is demonstrated numerically by considering a simulation of the measured Jones matrices of a transmission line with uniformly distributed but randomly oriented birefringence and polarization-dependent loss. The transmission-line model is a Monte-Carlo based simulation that uses 100 sections for each fiber realization. The amount of birefringence and PDL in each section is equal and selected in order to produce an average PDL of 2 dB and an average DGD of 10 ps. The conventional implementation of this model uses randomly generated rotations between sections to produce a fiber realization that is repeated a large number of times in order to examine the statistical properties of the stochastic transmission line. In contrast, the model used here numerically perturbs the initial fiber realization using zero-mean normally distributed rotations of the local birefringent and PDL vectors that are associated with each section. The simulation results presented here are for an evolution through 15,000 dynamic rotations for each of ten different widths of the perturbation probability density \(0.0625/m, \) where \(m = \{1, 1.5, 2, 2.5, 3, 3.5, 4, 5, 10, 20\}\). The different widths are used to examine a large range of rotation dynamics.

The evolving fiber realizations are used to determine the difference between the actual fiber state and that determined from simulated measurements that are acquired using a procedure that is similar to that used by commercial equipment. This procedure uses three linearly polarized launch states \(0^\circ, 45^\circ, \) and \(90^\circ\) with the resulting output SOP to calculate the Jones matrix. The state of the system evolved between launch states, thereby causing each launch state to capture a slightly different snapshot of a fiber realization. Figure 6 shows a scatter plot of the simulated measurements versus the average rotation experienced in the output SOP over the course of the launches that are needed to complete a measurement of the Jones matrix. A linear-regression analysis of this scatter plot yields a slope of 0.57 dB/degree, which indicates a correlation resulting from the dynamic artifact caused by the drifting system during the acquisition of the simulated measurement. Because this is a simulation of a measurement procedure, the instantaneous state of the transmission line is known. It is therefore possible to create a scatter plot from every instantaneous launch state that was used to create the previous figure. Figure 7 graphs the same simulation as Fig. 6 but uses the PDL from the instantaneous Jones matrices (resulting in a threefold increase in data points). In contrast to the simulated measurement that showed a correlation between the change in SOP and the “measured” PDL, Fig. 7, which has a linear-regression slope of \(4.6 \times 10^{-5} \) dB/degree, shows that this correlation is a measurement artifact and that the PDL measurements can be influenced by SOP dynamics.

It is important to note that PDL and SOP are independent quantities that have been shown to exhibit statistical behavior. This causes a loose grouping in the PDL versus change in SOP plot, but, as shown in the previous figures, can be used to determine the degree of correlation between the SOP and the measured PDL of a system. By considering the changing SOP to be a form of Brownian motion, significant information about the change in SOP can be derived from an undersampled SOP series as long as there is not angle ambiguity in the change of SOP for a given sampling rate. That is, as long as the angular deviation between adjacent SOP measurements is significantly less than \(\pi,\) the entire trajectory of the moving SOP does not need to be measured in order to characterize the motion. This allows the change in SOP and PDL correlation to be used as a practical metric of data robustness for systems that measure the Jones matrix of a transmission line.
5. APPLICATION OF ANALYSIS ON AN IN-SERVICE NETWORK

The Jones matrices of the network link under test are measured as a function of time and wavelength. Twelve measurements are collected in 0.05-nm spacing across the center 61.5 GHz of a 100-GHz-wide passband of a single WDM channel. These wavelength measurements are separated by approximately 5 s, which is the time required for the laser to be tuned to the next wavelength of interest and the entire Jones matrix to be determined by launching several states of polarized light and receiving the output polarizations. Therefore in terms of Fig. 3, \( \delta t = 5 \) s, \( \delta T = 60 \) s, and \( \delta \lambda = 0.05 \) nm. A measurement series is the set containing a single measure of the Jones matrix at each wavelength of interest. The time and frequency dependences of the Jones matrices are analyzed off line to determine the polarization-dependent characteristics of the network link. Measurement uncertainties include wavelength uncertainty in the tuning laser, noise that results from the detection system, and optical amplification, which are compounded during calculations of the Jones matrix and the PMD operator. The experimental system uses a tunable external-cavity semiconductor laser that has a wavelength uncertainty of \( \pm 0.005 \) nm in conjunction with an HP8509B polarization analyzer to measure the Jones matrix. This polarization analyzer has an uncertainty in the PDL of 0.1 dB and an uncertainty in the DGD of 300 fs when used with a wavelength step of 0.1 nm between measurements. For smaller wavelength steps, the measurement uncertainty of the DGD is expected to be slightly larger than this number. There is a maximum discernible DGD of 300 fs, which is caused by the uncertainty for rotations that are larger than a half-wave rotation between the measured Jones matrices at adjacent wavelengths.

The link under test is a multicomponent WDM network span consisting of an 80-km round-trip path of SMF-28 single-mode fiber, as shown in Fig. 8. The fiber of this network path is primarily underground except where bridge crossings are necessary. The test signal, denoted as \( \lambda 6 \) in the figure, was passively combined with existing WDM traffic at the testing node and transmitted within an available 100-GHz wavelength channel. The test signal was transparently looped back on the WDM multiplexers of the optical add–drop multiplexer at the return node. Along the transmission path, the signal also passed through four gain-clamped erbium-doped fiber amplifiers, two located at the midpoints of each direction and two others located on both trunk sides of the optical add–drop multiplexers at the return node. A passive 1 \( \times \) 2 tap followed by a WDM demultiplexer were used to separate the test signal from the network traffic at the measurement apparatus.

The analysis procedure of the raw data is as follows. First, the individual components of the measured Jones matrices at a constant wavelength are smoothed with a ten-point adjacent average filter such that Stirling’s scheme can be implemented. Adjacent measurements at a constant wavelength are spaced in 1-min intervals, which is the time required to complete a measurement series such that the Jones matrix is measured at that same wavelength. The PDL vector is determined for each measured Jones matrix, and the DGD and DAS vectors are calculated for isochronal Jones matrices using adjacent wavelengths (interpolated using Stirling’s central-difference scheme) and the Pauli-matrix equivalence technique. The 11 impairment results calculated for the 12 wavelengths are averaged for each measurement sequence and are shown in Fig. 9 along with measurements of the region’s wind speed for comparison. The wavelength and time-averaged impairments are \( \langle DGD \rangle = 7.09 \) ps, \( \langle DAS \rangle = 3.03 \) ps, and \( \langle PDL \rangle = 1.39 \) dB. The statistics of the SOP rotation are shown in Fig. 10 to have a mean of \( \langle \Delta SOP \rangle = 9.5^\circ \), which should have a dynamic artifact that is less than the uncertainty of the measurement apparatus as indicated by the data-interpolation scheme and isochronal-analysis simulations that are presented above.

The same analysis was conducted using the analysis method that removed data that had estimated errors values that exceeded 0.5 ps. The results of this analysis are qualitatively unchanged and have been reported elsewhere.

These measurements were also repeated for a separate link that consists of only optical fiber that is of the same age, type, and installation protocol as the metro link but follows a different path to a different return node. This fiber-only link has wavelength- and time-averaged im-

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**Fig. 7.** Scatter plot of the averaged instantaneous PDL determined from the dynamic Monte Carlo simulation. The independence of the output state of rotation demonstrates the independence of the dynamic evolution of the fiber realization and the actual PDL.

**Fig. 8.** Configuration of the WDM metro network that was analyzed for polarization-based impairments. The analysis system was at one node, passively multiplexed into the WDM trunk, separated again at the return node, and returned through a separate fiber in the same fiber bundle.
6. DISCUSSION

A few observations are readily apparent in Fig. 9. The first is the general correlation between the wind speed and the magnitude of each polarization-based impairment. While other field tests have shown sensitivity to temperature caused by the daily cycle, to our knowledge there has not been a previously published field measurement with sufficient time resolution to register changing weather patterns. The coincidence of the impairments and the wind is surprising because a direct coupling mechanism between the wind and the PDL, DGD, or DAS does not easily come to mind. It is important to recall that the PDL results are obtained from a single measurement of the Jones matrix, which for the implemented equipment is approximately 1 s, and yet there are variations in the PDL, as shown by Fig. 9. The most plausible explanation is that the system has a SOP that moves significantly in the time required to measure the Jones matrices. A scatter plot of the PDL versus SOP rotation experienced by time-adjacent and constant-wavelength Jones matrices is shown in Fig. 11. This is an illustration of the correlation between the SOP and the PDL in measured data, and it is most likely the result of a dynamic artifact, as outlined above.

It is important to note that the existence of a correlation is not sufficient evidence to determine cause and effect and that, while we have presented one physically reasonable scenario, these data do not rule out the possibility that there is a component or components that are coupled to the wind either directly through vibrations, such as the deflection of a bridge, or indirectly through a mechanism such as wind chill, or that the combination of gain-clamped amplifiers, variation in the SOP, and multiple WDM channels interact in such a way that polarization-dependent gain and birefringence interact to produce the observed results.

Measurements on this link were repeated four times over the course of 5 months, and in some cases the number of wavelengths was changed such that the time between Jones measurements at a specific wavelength was varied. The results of the linear regression between the PDL and SOP rotation occurring during the time required to repeat the measurement wavelength are summarized in Table 1. Even though these measurements are acquired in different months, and on two network links, the impairments of \( \langle \text{DGD} \rangle = 3.20 \text{ ps}, \langle \text{DAS} \rangle = 1.94 \text{ ps}, \langle \text{PDL} \rangle = 0.99 \text{ dB}, \) and \( \langle \text{sOP} \rangle = 8.07^\circ \).

Fig. 9. Magnitudes of the DGD, DAS, and PDL vectors are plotted along with the region's wind speed. Measurements of all quantities are collected with a resolution of approximately one minute. The correlation between the wind and the polarization-based impairments is a clear indication of the need to account for all possible complications when conducting link-assessment experiments.

Fig. 10. Statistics of the SOP rotation occurring in 1-min intervals during the course of the measurement window. This amount of rotation should be easily handled using an isochronal analysis and data interpolation.

Fig. 11. Scatter plot of the measured PDL and rotation of the SOP showing a correlation that indicates the influence of the average wind speed on measurements of the Jones matrices.
linear regression consistently shows evidence of a dynamic artifact, which casts uncertainty on the quality of the acquired data and any analysis derived from them. Since the SOP measurements are completed on a time scale that is between one and two orders of magnitude larger than the time required to complete a single Jones measurement, the cause of this correlation is indeterminable and can only be solved through measurements that occur on a much faster time scale. This lack of time resolution could be overcome through a redesign of the equipment that would allow the Jones matrices at a particular wavelength to be acquired on a millisecond time scale and could be further improved by simultaneous measuring of the Jones matrices at multiple wavelengths. An independent measure of the SOP rotation at a fixed wavelength would increase the complexity of the apparatus but would provide a criterion in which the fitness of a Jones-matrix measurement could be assessed and also used to determine the time-dependent rotation component of the complete SOP differential.

7. CONCLUSIONS

This paper identifies and provides solutions to three areas that can lead to a physically inaccurate assessment of an installed fiber-optic link. The presented solutions include a test for robustness in the measured data, a means to remove time-dependent rotations from the analysis, and two methods to analyze the measurement results in order to interpret physically significant results.

Measurements of a metro WDM network that is installed primarily underground showed significant sensitivity to the wind, which created dynamic artifacts that were much faster than what was expected. These results demonstrate the hazards that are present during the assessment of a fiber-optical network and the ease to which incorrect interpretations can be formed. To some degree, the physical limitations of multiple polarization-based impairments and dynamic polarization effects exist for all measurement approaches, and both need to be considered during every assessment of an installed transmission line.

**Table 1. Linear Regression of the Polarization-Dependent Loss and Rotation in the Output State of Polarization**

<table>
<thead>
<tr>
<th>Series</th>
<th>Slope (dB/degree)</th>
<th>Intercept (dB)</th>
<th>Relative Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instantaneous simulation</td>
<td>$7.62 \times 10^{-5}$</td>
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<td>$\infty$</td>
</tr>
<tr>
<td>Dynamic simulation</td>
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<td>1.62</td>
<td>1</td>
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<td>Metro-link series 1</td>
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<td>Metro-link series 2</td>
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<td>Fiber-link series 1</td>
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<td>0.26</td>
<td>60</td>
</tr>
</tbody>
</table>

*Relative time indicates the ratio between the time between Jones measurements and the time needed to acquire a single Jones measurement.*

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