Optimization of adiabaticity in coupled-waveguide devices using shortcuts to adiabaticity

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Conventional strategies to design adiabatic coupled-waveguide devices focus on optimizing the system adiabaticity but can only guarantee 100% efficiency at specific lengths. We establish a simple technique allowing the optimization of device adiabaticity and ensuring 100% coupling/conversion efficiency at any physically realizable length. Specifically, we use the shortcuts-to-adiabaticity technique to represent the system state precisely and engineer the system evolution to be as close to the adiabatic state as possible. Smooth parameters are derived for coupled-waveguide devices, which feature good robustness against wavelength and fabrication variations at the same time. The proposed device is verified with beam propagation simulations. © 2015 Optical Society of America

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Coupled-waveguide devices are widely used in integrated optics for power coupling [1], mode conversion [2], polarization rotation [3], mode multiplexing [4,5], etc. In addition to high coupling/conversion efficiency, developing devices with tolerance to variations in fabrication and wavelength is also a major technical challenge. In this regard, adiabatic devices have attracted lots of attention because they do not require strict fabrication control and are broadband. The operating principles of adiabatic (mode evolution)-based devices are based on the slow evolution of local waveguide mode (adiabatic mode) caused by gradual changes in the device geometry. When the adiabatic condition is met, coupling between adiabatic modes is small, leading to good device robustness against variations. On the other hand, adiabatic devices generally require longer lengths in order to meet the adiabatic condition and to obtain high coupling/conversion efficiency. The optimal design procedure of adiabatic coupled-waveguide devices has been proposed in different ways. One major strategy is to optimize the shape functions to minimize coupling between the adiabatic modes [2,6,7]. These approaches focus on optimizing the device adiabaticity, and the coupling/conversion efficiency will only be 100% at specific device lengths [1,8,9]. An adiabatic device that is guaranteed to have 100% efficiency at any physically realizable device length is thus very desirable.

“Shortcuts to adiabaticity” (STA) [10], originally developed in the context of quantum control, describe externally manipulated paths that lead to the same system state as an adiabatic process, but in a shorter time. The development of STA has lead to a series of new techniques to shorten the slow adiabatic process while keeping or enhancing system robustness against noise. These developments in quantum control are highly relevant to our goal of optimizing the adiabaticity in coupled-waveguide devices due to the analogies between quantum mechanics and wave optics [11], which allow one to manipulate light in waveguide structures based on their analogies with coherent quantum phenomena. By applying STA to coupled-waveguide devices, a series of waveguide-based devices such as mode converters [12,13], directional couplers [14–16], mode (de)multiplexers [17,18], and polarization rotators [19] have been proposed. A common feature of these devices is that they have good robustness against fabrication error or wavelength variations and are shorter than the conventional adiabatic designs. The reason is that STA allows one to design the most efficient system evolution from the desired initial state to the final state along a path that is robust against one or multiple errors. In particular, the invariant-based STA approach treats the system evolution using a set of decoupled system states that are equivalent to the eigenstates of the Lewis–Riesenfeld invariant [16,20,21]. This approach precisely describes the system evolution and can guarantee 100% coupling efficiency for arbitrary device lengths. The invariant-based approach also provides an efficient way to analyze system stability versus different types of noise using perturbation treatments [22–24], leading to the design of robust coupled-waveguide devices [16]. These approaches, while being robust against particular errors by design, do not guarantee adiabaticity. A design that is robust against a particular error might be highly susceptible to other sources of error. In this Letter, we optimize the adiabaticity of coupled-waveguide devices by designing the system evolution to be as close to the adiabatic state as possible using the decoupled system states, thus guaranteeing both adiabaticity
and 100% efficiency. Adiabatic devices designed with this new approach are now robust against various sources of errors, instead of only robust against specific errors by design [15,16,22–24]. This is achieved by expanding the phase of the decoupled system state using Fourier series. The central result of this work is that system adiabaticity can be achieved using only the third order expansion, resulting in very smooth shape functions that are highly suitable for coupled-waveguide devices.

The theory described in this Letter is applicable in general to weakly coupled waveguide structures described by the coupled-mode theory. For the convenience of derivation, we consider two waveguides, waveguide 1 and waveguide 2, placed in proximity with propagation constants \( \beta_1 \) and \( \beta_2 \). The refractive index or geometry of the two waveguides are allowed to vary along the propagation direction \( z \). Light is coupled into the device at \( z = 0 \) and out at \( z = L \). Under the scalar and paraxial approximation and assuming weak coupling [25], the changes in the guided-mode amplitudes in the individual waveguides \( |\Psi_j\rangle = [A_j, A_2]^T \) with propagation distance are described by the coupled-mode equation as [26]

\[
\frac{d}{dz}|\Psi\rangle = -i \begin{bmatrix} -\Delta \\ \Omega \end{bmatrix}|\Psi_0\rangle = -iH|\Psi\rangle,
\]

where \( \Omega \) (real) is the coupling coefficient, and \( \Delta = (\beta_1 - \beta_2)/2 \) describes the degree of mismatch between the waveguides. Replacing the spatial variation \( z \) with the temporal variation \( t \), Eq. (1) is equivalent to the time-dependent Schrödinger equation \( (\hbar = 1) \) describing the interaction dynamics of a two-state system driven by a coherent laser excitation, and \( H \) is the Hamiltonian. Solving for the eigenvectors of \( H \), we find two adiabatic modes, which are \( z \)-dependent superpositions of the unperturbed modes \( |1\rangle = [1, 0]^T \) and \( |2\rangle = [0, 1]^T \),

\[
|\Phi_+\rangle = \sin \Theta(z)|1\rangle + \cos \Theta(z)|2\rangle,
\]

\[
|\Phi_−\rangle = -\cos \Theta(z)|1\rangle + \sin \Theta(z)|2\rangle,
\]

where \( \Theta(z) = (1/2) \tan^{-1}(\Omega/\Delta) \). The two eigenvalues of \( H \) are

\[
e_{±} = \pm \sqrt{\Delta^2 + \Omega^2}.
\]

When the evolution is adiabatic, no coupling between the adiabatic modes occur. Mathematically, the adiabatic criterion requires that the coupling between the eigenvectors of \( H \) is negligible compared with the difference between the eigenvalues, i.e., [27]

\[
\frac{\langle \Phi_+ | \Phi_− \rangle}{|e_+ - e_-|} \ll 1, \tag{5}
\]

where the dot denotes the derivative with respect to \( z \). The goal in this work is to obtain 100% conversion from an unperturbed mode to the other by properly selecting \( \Omega \) and \( \Delta \), while optimizing system adiabaticity represented by Eq. (5).

Following the invariant-based STA scheme, the solution of Eq. (1) can be described by the following decoupled system states [16]:

\[
|\Psi_+\rangle = \cos(\theta/2)e^{-i\phi/2}|1\rangle + \sin(\theta/2)e^{i\phi/2}|2\rangle, \tag{6}
\]

\[
|\Psi_−\rangle = \sin(\theta/2)e^{-i\phi/2}|1\rangle - \cos(\theta/2)e^{i\phi/2}|2\rangle, \tag{7}
\]

where

\[
\dot{\theta} = \Omega \sin \phi, \tag{8}
\]

\[
\dot{\phi} = \Delta + \Omega \cos \phi \cot \theta. \tag{9}
\]

To describe 100% coupling between two coupled waveguides by Eq. (1), the initial and final states of the system are set as \( |\Psi(0)\rangle = |2\rangle \) and \( |\Psi(L)\rangle = |1\rangle \), respectively. The state evolution may be parameterized according to one of the decoupled states \( |\Psi_+\rangle \) or \( |\Psi_−\rangle \). By using \( |\Psi\rangle = |\Psi_+\rangle \), the boundary conditions

\[
\theta(0) = 0, \quad \theta(L) = \pi \tag{10}
\]
guarantee the desired initial and final states. There is still much freedom to design \( \theta \) and \( \phi \), except for the boundary conditions. The freedom allows one to optimize system adiabaticity by designing the system evolution to follow one of the eigenvectors \( |\Phi_+\rangle \) as closely as possible.

Following [15], we choose a smooth function satisfying Eq. (10):

\[
\theta = \frac{\pi}{2} \left[ 1 + \sin \left( \frac{\pi(2z - L)}{2L} \right) \right]. \tag{11}
\]

Applying l'Hôpital’s rule to Eq. (9) repeatedly, we find additional boundary conditions

\[
\phi(0) = \pi/2, \quad \phi(L) = \pi/2. \tag{12}
\]

Full adiabaticity can be obtained if we further require \( \phi(z) = 0 \) during the evolution (except for the boundaries) so that the trajectory of \( |\Psi_+\rangle \) and \( |\Phi_+\rangle \) would completely overlap on the Bloch sphere. However, a closer inspection of Eq. (8) shows that \( \phi(z) = 0 \) leads to an infinitely large \( \Omega \), which is physically unrealizable. Instead, we set \( \phi \) to a small value \( c \) that is directly related to the maximum obtainable coupling coefficient in the coupled-waveguide system under consideration. The optimization scheme is now reduced to designing \( \theta(z) \) such that it satisfies

\[
\phi(0) = \phi(L) = \pi/2 \quad \phi(z) = c. \tag{13}
\]

To obtain a smooth \( \phi \) that leads to a smooth set of \( \Omega \) and \( \Delta \), we expand the target \( \phi \) function in Eq. (13) using Fourier series with \( \sigma \)-approximation to eliminate the Gibbs phenomenon associated with the discontinuities [28]

\[
\phi(z) = \frac{\pi}{2} + \left( \frac{\pi}{2} - c \right) \sum_{k=1}^{n} a_k \sin \left( \frac{\pi k z}{L} \right) \quad (n \text{ is odd}). \tag{14}
\]

In this work, we limit the maxima of \( \Omega \) to 16 mm−1 in the device we will subsequently design, which corresponds to \( c = 0.3128 \) in Eq. (13). The results of expansion coefficients for the first few orders are given in Table 1, and the corresponding \( \phi \) functions are plotted in Fig. 1. The shape of \( \phi \) approaches the conditions outlined in Eq. (13) as the expansion order is increased.

The corresponding \( \Omega \) and \( \Delta \) functions are then obtained using Eqs. (8) and (9) and illustrated in Fig. 2. We note that higher order expansions lead to \( \Omega \) and \( \Delta \) that oscillate more and more, corresponding to large bends and width variations in devices, as will be explained later. These effects will introduce more loss in the transmission and have to be considered in device design.

Next, we examine the device adiabaticity by calculating the adiabatic criterion in Eq. (5) for different orders of \( \Omega \) and \( \Delta \), and
the result is shown in Fig. 3. It is clear that device adiabaticity can be optimized by using only the third-order expansion, which corresponds to a smooth set of Ω and Δ (blue dashed lines) in Fig. 2. The third order and above Ω and Δ functions in Fig. 2 thus represent optimized coupling-coefficient and waveguide-mismatch functions for adiabatic coupled-waveguide devices described by Eq. (1), and 100% efficiency is guaranteed by the boundary conditions we set in Eq. (10).

We now illustrate the design of an adiabatic coupler using the third order coefficients (blue dashed lines) in Fig. 2 and perform beam-propagation method (BPM) simulations to verify the designs. The BPM code used in the simulations solves the scalar and paraxial wave equation using the finite difference scheme with the transparent boundary condition [25]. The polymer channel waveguide structure is the same as the one considered in [15], where 3 μm thick SiO₂ (n = 1.46) on a Si (n = 3.48) wafer is used for the bottom cladding layer, the core consists of a 2.4 μm layer of BCB (n = 1.53), and the upper cladding is epoxy (n = 1.50). The waveguide center-to-center spacing D and width difference δW are adjusted along the propagation direction to satisfy the designed set of Ω and Δ functions. The relation between Δ and δW can be approximated by a linear relation [6], and the relation between Ω and D in a symmetric coupler is well fitted by the exponential relation \( \Omega = \Omega_0 \exp(-\gamma(D - D_0)) \) [25]. We also assume that the exponential relation can be used to obtain an estimation of the coupling coefficient in the asymmetric coupler [6].

The coupler is designed at a length of 1 mm, and the default waveguide width is W₀ = 2 μm. Using the exponential relation between Ω and D and the linear relation between Δ and δW, we obtain the corresponding device parameters in Fig. 4, and the geometry of the designed adiabatic coupler is shown in Fig. 5. We excite the bottom waveguide by its unperturbed mode at z = 0 with an input wavelength of 1.55 μm, and the BPM result is shown in the same figure.

Remarkably, power is 100% coupled to the top waveguide at the arbitrarily chosen device length of 1 mm, which cannot be guaranteed with conventional adiabatic schemes. For a given \( \phi \) function, we can see from Eq. (8) that a shorter device length (larger \( \theta \)) leads to a large Ω. Therefore, the minimum device length is limited by the maximally achievable coupling coefficient in the chosen waveguide system.

We then look at the device robustness against wavelength and fabrication variations. Figure 6 shows the simulated wavelength dependence of the coupling efficiency. It can be seen that for a wide range from 1.2 to 2.0 μm, the coupling efficiency is larger than 90%. Next, the fabrication tolerance is investigated

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**Table 1. Expansion Coefficients for \( \phi \) in Eq. (14)**

<table>
<thead>
<tr>
<th>Order</th>
<th>( a_1 )</th>
<th>( a_3 )</th>
<th>( a_5 )</th>
<th>( a_7 )</th>
</tr>
</thead>
<tbody>
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<td>( n = 1 )</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( n = 3 )</td>
<td>-1.1250</td>
<td>-0.1250</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( n = 5 )</td>
<td>-1.2282</td>
<td>-0.2717</td>
<td>-0.0489</td>
<td>0</td>
</tr>
<tr>
<td>( n = 7 )</td>
<td>-1.2378</td>
<td>-0.3320</td>
<td>-0.1195</td>
<td>-0.0253</td>
</tr>
</tbody>
</table>

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**Fig. 1.** Calculated \( \phi \) functions for different orders of Fourier series expansion with \( \sigma \) approximation (with the parameters of Table 1, \( L = 1 \) mm).

**Fig. 2.** Calculated Ω and Δ functions corresponding to different orders of \( \phi \) (\( L = 1 \) mm).

**Fig. 3.** Adiabatic criterion defined in Eq. (5) calculated for different orders of \( \phi \).

**Fig. 4.** Device parameters for the designed adiabatic coupler. (a) Center-to-center spacing D. (b) Width difference δW.

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at the operating wavelength of 1.55 μm by adding a constant error to the waveguide spacing in the simulation. The simulation result is shown in Fig. 7. It can be seen that for a spacing error as large as −1.5 to 0.75 μm, the coupling efficiency is greater than 90%. The good robustness of the device indicates that the adiabaticity of the device is indeed optimized with our scheme. We also note that BPM simulations with devices designed using higher order expansions show similar robustness as the one using the third order expansion, agreeing with the adiabaticity analysis in Fig. 3.

We have put forward a scheme to optimize the adiabaticity of coupled-waveguide devices using shortcuts to adiabaticity. Decoupled system states that are eigenvectors of Lewis–Riesenfeld invariants are used to precisely engineer the system evolution. The adiabaticity is optimized by engineering the evolution of the system state to be close to the adiabatic state, while the boundary conditions set on the parameters of the decoupled states ensure 100% efficiency. The result is obtained by simply expanding the target phase function of the decoupled system state using Fourier series, without the need to optimize or nullify any specific functions. The resulting design shows good robustness to errors in Ω and Δ simultaneously. The result could also find applications in designing the time-dependent Rabi frequencies and detunings needed to control two-state quantum systems governed by the time-dependent Schrödinger equation.

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**REFERENCES**