Robust synchronization of drive–response chaotic systems via adaptive sliding mode control

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Abstract

A robust adaptive sliding control scheme is developed in this study to achieve synchronization for two identical chaotic systems in the presence of uncertain system parameters, external disturbances and nonlinear control inputs. An adaptation algorithm is given based on the Lyapunov stability theory. Using this adaptation technique to estimate the upper-bounds of parameter variation and external disturbance uncertainties, an adaptive sliding mode controller is then constructed without requiring the bounds of parameter and disturbance uncertainties to be known in advance. It is proven that the proposed adaptive sliding mode controller can maintain the existence of sliding mode in finite time in uncertain chaotic systems. Finally, numerical simulations are presented to show the effectiveness of the proposed control scheme.

1. Introduction

Since Pecora and Carroll [1] introduced a PC method to synchronize two identical chaotic systems with different initial conditions in 1990, chaos synchronization has received increasing attention and has been investigated widely due to its potential application in wide areas of physics and engineering sciences, such as secure communication, information processing, biological systems, and chemical reaction [2–4]. Many effective control methods [5–8] have been proposed to achieve chaos synchronization, such as linear and nonlinear feedback controls. However, most researches on chaos synchronization focused on certain chaotic systems without consideration of parameter variation uncertainty and external disturbance perturbation. But in practical situations, many chaotic systems are inevitably affected by parameter variations and external disturbances. Moreover, some or all of the system parameters and external disturbance uncertainties are unknown or variable from time to time. Therefore, investigation of system parameter variations and external disturbance perturbations in synchronization between drive and response chaotic systems has become an interesting and important research topic in recent years. In the work [9], an adaptive control law with single-state variable feedback was derived and applied to achieve the state synchronization of two identical Lorenz systems. An active sliding mode control was proposed by Zhang and Ma [10] to synchronize chaotic systems with parameter perturbation. Zhang et al.

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[11] also presented a sliding mode control to resolve the conquer synchronization problem in noise-perturbed chaotic systems. In the work [12], an intermittent parametric adaptive control method was studied to synchronize two logistic maps, and the corresponding sufficient conditions for synchronization are drawn. Based on Lyapunov stabilization theory, Huang et al. [13] proposed an adaptive controller with parameters identification for synchronizing a class of chaotic systems with unknown parameters. Park [14] developed a nonfragile controller using the Lyapunov functional technique combined with LMI technique to achieve synchronization problem of a class of chaotic systems with controller gain variations.

Owing to physical limitations, there always exist nonlinear effects in the control actuators. Neglecting the effect of input nonlinearities usually results in degeneration upon system performance. In addition to the problem of parameter variation uncertainty and external disturbance perturbation, the nonlinear input effect is also discussed in this paper. Sliding mode control is a well-known approach with its robustness against uncertainties, disturbances, and unmodeled dynamics. Therefore, in this paper, an adaptive sliding mode control is proposed to achieve the synchronization problem for two identical chaotic systems in the presence of uncertain system parameter variation, external disturbance perturbation, and nonlinear control inputs. Based on the Lyapunov stability theory, an adaptation law is given to estimate the upper-bound values of system uncertainties recursively. Consequently, a robust adaptive sliding mode controller can be constructed by applying the adaptation law: the existence of sliding mode can be maintained even without knowing the upper-bounds of parameter and disturbance uncertainties. Finally, numerical simulations are presented to show the effectiveness and robustness of the proposed control scheme.

2. System description and problem formulation

Consider the following chaotic system described by

\[ \dot{x} = Ax + f(x) + g(t), \]

where \( x(t) \in \mathbb{R}^n \) is a \( n \)-dimensional state vector of the system, \( A \in \mathbb{R}^{n \times n} \) is the matrix of the system parameter, \( g(t) \in \mathbb{R}^n \) is the external input signal and \( f(\cdot) : \mathbb{R}^n \to \mathbb{R}^n \) is a continuous nonlinear function vector satisfying the following global Lipschitz condition,

\[ \| f(x) - f(y) \| \leq \rho \| x - y \|, \quad \forall x, y \in \mathbb{R}^n, \]

where \( \| \cdot \| \) denotes the Euclidean norm and \( \rho \) is the Lipschitz constant. As we know, many chaotic systems investigated are based on form (1), such as Murali–Lakshmanan–Chua (MLC) system [15] and Genesio system [16]. The problem discussed in this paper concerns the synchronization problem of system (1) using the drive–response configuration. Here, the system (1) is considered as a drive system. A response system with the same form of Eq. (1) with the nonlinear control input vector \( n(\cdot) : \mathbb{R}^n \to \mathbb{R}^n \) and external disturbance \( d(t) \in \mathbb{R}^n \) is introduced as follows:

\[ \dot{y} = A_1 y + f(y) + g(t) + n(u) + d(t), \]

where \( y(t) \in \mathbb{R}^n \) is the state vector of the response system, \( A_1 \in \mathbb{R}^{n \times n} \) is an unknown constant system parameter matrix, \( n(u) = [n_1(u_1), n_2(u_2), \ldots, n_n(u_n)]^T \in \mathbb{R}^n \) is a continuous nonlinear function vector with \( n(0) = 0 \), \( u = [u_1, u_2, \ldots, u_n]^T \in \mathbb{R}^n \) is a control vector, \( d(t) \in \mathbb{R}^n \) is an external disturbance. The response system has the same structure as the drive system, except that it is subjected to unknown parameter variations and external disturbance uncertainties. Before proceeding with the main results of this paper, the following assumptions, which specify the class of uncertain response systems are made.

Assumption 1. Nonlinear input functions \( n_i(u_i) \), \( i = 1, \ldots, n \), are sector bounded by \( u_i \), \( i = 1, \ldots, n \), respectively. It yields positive constants \( c_{i1}, i = 1, \ldots, n \) and \( c_{i2}, i = 1, \ldots, n \), such that the following conditions are satisfied. \( c_{i1} \leq \frac{\| n_i \|}{u_i} \leq c_{i2} \), for \( i = 1, \ldots, n \).

Assumption 2. Uncertain system parameter error matrix and external disturbance vector are norm-bounded. It means that there exists two positive constants \( \mu_1 \) and \( \mu_2 \) large enough such that \( \| A_1 - A \| \leq \mu_1 \) and \( \| d(X, t) \| \leq \mu_2 \).

From Assumption 1, it can be obtained straightforward that

\[ c_{i1} u_i^2(t) \leq u_i(t) n_i(u_i(t)) \leq c_{i2} u_i^2(t), \quad i = 1, \ldots, n. \]

Then, we have

\[ c_{i1} u_i^2(t) + \cdots + c_{i1} u_i^2(t) \leq u_i(t) n_i(u_i(t)) + \cdots + u_n(t) n_n(u_n(t)) \leq c_{i2} u_i^2(t) + \cdots + c_{i2} u_n^2(t). \]
Then, the dynamic equation of synchronization error can be expressed as

\[ c_1 u^T(t)u(t) \leq u^T(t)n(u(t)) \leq c_2 u^T(t)u(t). \]

In this paper, the synchronization problem is how to design a suitable control input \( u(t) \) for different initial conditions between drive and response systems, such that it can synchronize the states of both the drive and the response systems in the presence of uncertain system parameters, external disturbances, and nonlinear control inputs.

3. Controller design

In this paper, we consider the synchronization of a class of uncertain chaotic systems in the drive–response framework. Due to the existence of system parameter and external disturbance uncertainties in error dynamics, the application of a robust controller is unavoidable. In this paper, we propose a sliding mode controller, which is of inherent advantages of easy realization, fast response, good transient performance, and insensitive to variations in system parameters and external disturbances.

3.1. Sliding mode control

Let us define the state error between the response system and drive system as

\[ e = y - x. \]

Then, the dynamic equation of synchronization error can be expressed as

\[ \dot{e} = Ae + f(y) - f(x) + (A_1 - A)y + n(u) + d(t). \]

The objective of this paper is that for any given drive system (1) and response system (2), a sliding mode controller is designed in spite of the unknown parameter and external disturbance uncertainties in the response system, such that the asymptotical stability of the resulting error dynamical system (5) can be achieved as \( \|e(t)\| \to 0 \) as \( t \to \infty \).

As a result, using a sliding mode control method to synchronize drive–response chaotic systems involves two major stages: (1) selection of an appropriate switching surface for the desired sliding motion; and (2) design of a sliding mode control law that brings any orbit in phase space to the switching surface and then achieves the synchronization of drive–response chaotic systems even in the presence of parametric and disturbance uncertainties.

For the sliding mode controller design in this paper, we need to select an appropriate switching surface with integral-operation, such that the sliding motion on the manifold has the desired properties. The sliding mode controller is designed as follows:

\[ S(t) = C\left[ e(t) - \int_0^t (A + K)e(\tau) \, d\tau \right] = 0, \]

where \( C \in \mathbb{R}^{n \times n} \) is a full rank constant matrix and \( K \in \mathbb{R}^{n \times n} \) is an error feedback gain matrix to be designed. To ensure the occurrence of sliding motion, the proposed control law is given by

\[ u = -\frac{C^TS}{\|C^TS\|} \cdot \frac{\gamma}{c_1} (\rho\|e\| + \mu_1\|y\| + \mu_2 + \|Ke\| + \eta), \]

where \( \gamma > 1 \) and \( \eta > 0 \) are two constants.

Theorem 1. Consider the error dynamical system (5) with parameter and disturbance uncertainties, the error state trajectory converges to the sliding surface \( S(t) = 0 \) if the sliding mode control law (7) is applied.

Proof. For simplification in the proof, we define

\[ \phi_1(x,y,t) = \frac{\gamma}{c_1} (\rho\|e\| + \mu_1\|y\| + \mu_2 + \|Ke\| + \eta). \]

Consider a Lyapunov function

\[ V = \frac{1}{2} S^TS. \]
Then, its first derivative along the error dynamical system (5) is
\[
\dot{V} = S^T \dot{S} = S^T C \dot{e} - (A + K) e = S^T C [f(y) - f(x) + (A_1 - A)y + n(u) + d - Ke]
\]
\[
\leq ||C^T S|| ||f(y) - f(x)|| + ||A_1 - A|| ||y|| + ||d|| + ||Ke|| + S^T C n(u)
\]
\[
\leq ||C^T S|| \left( ||\rho e|| + ||\mu_1 || ||y|| + ||\mu_2 + ||Ke|| \right) + S^T C n(u).
\]
From inequality (3) and Eqs. (7) and (8), one can obtain
\[
u^T n(u) = -\frac{S^T C}{||S^T C||} \phi_1(x, y, t)n(u) \geq c_1 u^T u = c_1 \frac{S^T C}{||C^T S||} \phi_1(x, y, t) \frac{C^T S}{||C^T S||} \phi_1(x, y, t) = c_1 \phi_1^2(x, y, t).
\]
From inequality (10), it yields that
\[-\frac{S^T C}{||S^T C||} \phi_1(x, y, t)n(u) \geq c_1 \phi_1^2(x, y, t).
\]
Then, we can obtain
\[S^T C n(u) \leq -c_1 ||S^T C|| \phi_1(x, y, t).
\]
Taking inequality (11) into inequality (9) we have
\[\dot{V} \leq ||C^T S|| \left( ||\rho e|| + ||\mu_1 || ||y|| + ||\mu_2 + ||Ke|| \right) - c_1 ||C^T S|| \phi_1(x, y, t)
\]
\[
= ||C^T S|| \left( ||\rho e|| + ||\mu_1 || ||y|| + ||\mu_2 + ||Ke|| \right) - \gamma ||C^T S|| \left( ||\rho e|| + ||\mu_1 || ||y|| + ||\mu_2 + ||Ke|| \right)
\]
\[= (1 - \gamma) ||C^T S|| \left( ||\rho e|| + ||\mu_1 || ||y|| + ||\mu_2 + ||Ke|| \right) - \gamma ||C^T S|| \leq -\gamma ||C^T S|| \leq 0.
\]
It is concluded that the reaching condition $S^T \dot{S} \leq 0$ is always maintained. Thus the proof is achieved completely. \(\Box\)

3.2. Adaptive sliding mode control

In this subsection, an adaptation law is applied to construct the proposed adaptive sliding mode controller such that the existence of sliding mode can be maintained even without knowing the upper-bounds of parameter and disturbance uncertainties. To ensure the occurrence of the sliding motion, an adaptive control law is proposed as
\[u = -\frac{C^T S}{||C^T S||} \frac{\gamma}{c_1} (\tilde{\beta}_1 ||e|| + \tilde{\beta}_2 ||y|| + \tilde{\beta}_3 + ||Ke|| + \eta),\]
where $\tilde{\beta}_1$, $\tilde{\beta}_2$ and $\tilde{\beta}_3$ are the estimated values of unknown constants $\beta_1 = \frac{c}{\gamma}$, $\beta_2 = \frac{a}{\gamma}$, and $\beta_3 = \frac{b}{\gamma}$, respectively. These estimated values will be adapted according to the following adaptation laws
\[\dot{\tilde{\beta}}_1 = \frac{\gamma}{k_1} ||e|| ||C^T S||, \quad \tilde{\beta}_1(0) = \tilde{\beta}_{10},\]
\[\dot{\tilde{\beta}}_2 = \frac{\gamma}{k_2} ||y|| ||C^T S||, \quad \tilde{\beta}_2(0) = \tilde{\beta}_{20},\]
\[\dot{\tilde{\beta}}_3 = \frac{\gamma}{k_3} ||C^T S||, \quad \tilde{\beta}_3(0) = \tilde{\beta}_{30},\]
where $k_1$, $k_2$ and $k_3$ are arbitrary positive constants, $\tilde{\beta}_{10}$, $\tilde{\beta}_{20}$ and $\tilde{\beta}_{30}$ are the bounded initial values of $\tilde{\beta}_1$, $\tilde{\beta}_2$ and $\tilde{\beta}_3$, respectively. Now define the parameter estimation errors as
\[\hat{\beta}_1 = \beta_1 - \tilde{\beta}_1,\]
\[\hat{\beta}_2 = \beta_2 - \tilde{\beta}_2,\]
\[\hat{\beta}_3 = \beta_3 - \tilde{\beta}_3.\]
Since $\beta_1 = \frac{c}{\gamma}$, $\beta_2 = \frac{a}{\gamma}$ and $\beta_3 = \frac{b}{\gamma}$ are existing constants, we have
\[\dot{\hat{\beta}}_1 = -\frac{\gamma}{k_1} ||e|| ||C^T S||,\]
\[\dot{\hat{\beta}}_2 = -\frac{\gamma}{k_2} ||y|| ||C^T S||,\]
\[\dot{\hat{\beta}}_3 = -\frac{\gamma}{k_3} ||C^T S||.\]
In the following, the proposed adaptive sliding mode control scheme in Eqs. (12)–(15) will be proved to be able to drive the error dynamic system (5) onto the sliding surface \( S(t) = 0 \).

**Theorem 2.** Consider the error dynamical system (5) with unknown parameter and disturbance uncertainties, this system is controlled by the adaptive sliding mode controller (12) with adaptation laws (13)–(15). Then the error state trajectory converges to the sliding surface \( S(t) = 0 \).

**Proof.** Consider a Lyapunov function as the following form:

\[
V = \frac{1}{2} \left( S^T S + k_1 \dot{\hat{b}}_1^2 + k_2 \dot{\hat{b}}_2^2 + k_3 \dot{\hat{b}}_3^2 \right).
\]

The time derivative of \( V \) along the error dynamical system (5) is

\[
\dot{V} = S^T \dot{S} + k_1 \dot{\hat{b}}_1 \ddot{\hat{b}}_1 + k_2 \dot{\hat{b}}_2 \ddot{\hat{b}}_2 + k_3 \dot{\hat{b}}_3 \ddot{\hat{b}}_3
\]

\[
\leq \|C^T S\| (\|\rho\| \|e\| + \mu_1 \|y\| + \mu_2 + \|Ke\|) + S^T C \dot{u} + k_1 \ddot{\hat{b}}_1 \ddot{\hat{b}}_1 + k_2 \ddot{\hat{b}}_2 \ddot{\hat{b}}_2 + k_3 \ddot{\hat{b}}_3 \ddot{\hat{b}}_3
\]

\[
\leq \|C^T S\| (\gamma \|\rho\| \|e\| + \gamma \|\beta_2\| \|y\| + \gamma \beta_3 + \|Ke\|) + S^T C \dot{u} + k_1 \ddot{\hat{b}}_1 \ddot{\hat{b}}_1 + k_2 \ddot{\hat{b}}_2 \ddot{\hat{b}}_2 + k_3 \ddot{\hat{b}}_3 \ddot{\hat{b}}_3. \tag{22}
\]

For simplicity, we set

\[
\phi_2(x,y,t) = \frac{\gamma}{c_1} (\ddot{\hat{b}}_1 \|e\| + \ddot{\hat{b}}_2 \|y\| + \ddot{\hat{b}}_3 + \|Ke\| + \eta). \tag{23}
\]

Similar to the derivation in the previous proof, it yields

\[
S^T C \dot{u} \leq -c_1 \|S^T C\| \phi_2(x,y,t). \tag{24}
\]

From inequalities (22) and (24), we can obtain

\[
\dot{V} \leq \|C^T S\| (\gamma \|\rho\| \|e\| + \gamma \|\beta_2\| \|y\| + \gamma \beta_3 + \|Ke\| - c_1 \|S^T C\| \phi_2(x,y,t) + \ddot{\hat{b}}_1 \ddot{\hat{b}}_1 + \ddot{\hat{b}}_2 \ddot{\hat{b}}_2 + \ddot{\hat{b}}_3 \ddot{\hat{b}}_3)
\]

\[
= \|C^T S\| (\gamma \|\rho\| \|e\| + \gamma \|\beta_2\| \|y\| + \gamma \beta_3 + \|Ke\| - \|C^T S\| (\ddot{\hat{b}}_1 \|e\| + \ddot{\hat{b}}_2 \|y\| + \ddot{\hat{b}}_3 + \|Ke\| + \eta) + \ddot{\hat{b}}_1 \ddot{\hat{b}}_1 + \ddot{\hat{b}}_2 \ddot{\hat{b}}_2 + \ddot{\hat{b}}_3 \ddot{\hat{b}}_3)
\]

\[
= (1 - \gamma) \|C^T S\| \|Ke\| + \ddot{\hat{b}}_1 (\gamma \|C^T S\| \|e\| + \ddot{\hat{b}}_1) + \ddot{\hat{b}}_2 (\gamma \|C^T S\| \|y\| + \ddot{\hat{b}}_2) + \ddot{\hat{b}}_3 (\gamma \|C^T S\| + \ddot{\hat{b}}_3) - \gamma \eta \|C^T S\| + k_1 \ddot{\hat{b}}_1 \ddot{\hat{b}}_1 + k_2 \ddot{\hat{b}}_2 \ddot{\hat{b}}_2 + k_3 \ddot{\hat{b}}_3 \ddot{\hat{b}}_3
\]

\[
= (1 - \gamma) \|C^T S\| \|Ke\| + \ddot{\hat{b}}_1 (\gamma \|C^T S\| \|e\| + \ddot{\hat{b}}_1) + \ddot{\hat{b}}_2 (\gamma \|C^T S\| \|y\| + \ddot{\hat{b}}_2) + \ddot{\hat{b}}_3 (\gamma \|C^T S\| + \ddot{\hat{b}}_3) - \gamma \eta \|C^T S\|. \tag{25}
\]

Inserting Eqs. (19)–(21) into the right hand of inequality, this yields

\[
\dot{V} \leq (1 - \gamma) \|C^T S\| \|Ke\| - \gamma \eta \|C^T S\| \leq -\gamma \eta \|C^T S\| \leq 0. \tag{26}
\]

From Eqs. (13)–(15), (19)–(21), it can be seen that \( \ddot{\hat{b}}_1 = -\ddot{\hat{b}}_1 = -\frac{1}{\tau_1} \|e\| \|C^T S\| \leq 0, \ddot{\hat{b}}_2 = -\ddot{\hat{b}}_2 = -\frac{1}{\tau_2} \|y\| \|C^T S\| \leq 0 \) and \( \ddot{\hat{b}}_3 = -\ddot{\hat{b}}_3 = -\frac{1}{\tau_3} \|C^T S\| \leq 0 \) and existing a finite time \( t_1 \) such that \( \ddot{\hat{b}}_1(t_1) = \ddot{\hat{b}}_1(t_1) \leq 0, \ddot{\hat{b}}_2(t_1) = \ddot{\hat{b}}_2(t_1) \leq 0 \) and \( \ddot{\hat{b}}_3(t_1) = \ddot{\hat{b}}_3(t_1) \leq 0 \). We then have

\[
\ddot{\hat{b}}_1(t) \ddot{\hat{b}}_1(t) \geq 0, \forall t \geq t_1, \tag{27}
\]

\[
\ddot{\hat{b}}_2(t) \ddot{\hat{b}}_2(t) \geq 0, \forall t \geq t_1, \tag{28}
\]

\[
\ddot{\hat{b}}_3(t) \ddot{\hat{b}}_3(t) \geq 0, \forall t \geq t_1. \tag{29}
\]

From inequalities (26)–(29), it concluded that there exists a finite time \( t_1 \) such that for all \( t \geq t_1 \), the reach condition \( S^T \ddot{S} < 0 \) is maintained. Thus the proof is \( S^T \ddot{S} \leq 0 \) achieved completely.

**4. Numerical simulations**

In this section, some numerical simulations are given to illustrate the performance of the proposed adaptive synchronization for uncertain drive–response chaotic systems by using MATLAB ODE113 solver. Here we consider an MLC circuit [17] expressed by

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
\sigma & -\beta
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
-f(x_1) \\
0
\end{bmatrix} +
\begin{bmatrix}
g_1(t) \\
g_2(t)
\end{bmatrix}.
\]

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where $\sigma > 0$, $\beta > 0$ and $f(\cdot)$ is a piecewise linear function in the following form:

$$f(x_1) = bx_1 + 0.5(a - b)(|x_1 + 1| - |x_1 - 1|)$$

with $a < b < 0$. Hence, a response system can be obtained as

$$\begin{bmatrix}
    \dot{y}_1 \\
    \dot{y}_2
\end{bmatrix} =
\begin{bmatrix}
    0 & 1 \\
    -\bar{a} & -\bar{b}
\end{bmatrix}
\begin{bmatrix}
    y_1 \\
    y_2
\end{bmatrix} +
\begin{bmatrix}
    -f(y_1) \\
    0
\end{bmatrix} +
\begin{bmatrix}
    g_1(t) \\
    g_2(t)
\end{bmatrix} +
\begin{bmatrix}
    n_1(u_1) \\
    n_2(u_2)
\end{bmatrix} +
\begin{bmatrix}
    d_1(t) \\
    d_2(t)
\end{bmatrix}$$

In this simulation, the parameters of MLC circuit are selected as $\sigma = 1.0$, $\beta = 1.015$, $a = -1.02$, and $b = -0.55$. External input signals are $g_1(t) = 5 \sin(13t) + 10 \sin(27t)$ and $g_2(t) = 10 \sin(15t) + 10 \sin(23t)$. For the response system, unknown parameters, nonlinear inputs, and external disturbances are taken as $\bar{a} = 0.8 - 0.2 \sin(0.1t)$, $\bar{b} = 0.5 - 0.1 \sin(0.05t)$, $n_1(u_1) = [2 - 0.5 \sin(u_1)]u_1$, $n_2(u_2) = [2 + 0.3 \sin(u_2)]u_2$, $d_1(t) = 0.1 \sin(10t)$, and $d_2(t) = 0.2 \sin(20t)$, respectively. The initial states of drive–response systems are $\hat{x}_1(0) = [0 \ 0]$ and $\hat{y}_1(0) = [1 \ 1]$.

For the adaptive sliding mode control scheme, design parameters are chosen as follows: $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $K = \begin{bmatrix}
    -5 & -1 \\
    1 & -3.985
\end{bmatrix}$, $\bar{\chi}_1 = \bar{\chi}_2 = \bar{\chi}_3 = 10$, $\bar{\chi}_4 = 5$ and $\eta = 0.1$.

Simulation results are shown in Figs. 1–4. Figs. 1 and 2 show the state trajectories of drive and response systems. Fig. 3 depicts the synchronization errors of state variables between drive and response systems. The switching functions responses of the overall system are shown in Fig. 4. The simulation shows that the proposed adaptive sliding mode con-
controller works well for the synchronization of drive–response chaotic systems with system parameter variations and external disturbance uncertainties.

5. Conclusions

In this paper, the design problem of adaptive sliding mode controller for synchronization of a class of drive–response chaotic systems with parameter variation uncertainty, external disturbance perturbation, and control input nonlinearity is studied using the Lyapunov method. The proposed control scheme can be implemented without requiring the bounds of unknown parameter and disturbance uncertainties to be known in advance. Numerical results show that the proposed control scheme is very effective and robust against system uncertainties.

References