RHEOLOGICAL CHARACTERISTICS FOR THIN FILM ELASTOHYDRODYNAMIC LUBRICATION WITH NON-NEWTONIAN LUBRICANTS

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ABSTRACT

The modified Reynolds equation for power law fluid is derived from the viscous adsorption theory for thin film elastohydrodynamic lubrication (TFEHL). The differences between classical non-Newtonian EHL and non-Newtonian TFEHL are discussed. Results show that the proposed model can reasonably calculate the pressure distribution, the film thickness, the velocity distribution and the average viscosity under thin film lubrication. The thickness ($\delta$), the viscosity ($m_1$), and the flow index ($n_1$) of the adsorption layer influence significantly the lubrication characteristics of the contact conjunction. Furthermore, the film thickness increases with the increase of $n_1$, and the film thickness affected by $m_1$ is greater than that affected by $n_1$, but the effect of $n_1$ produces a very small difference in the pressure distributions. In addition, the greater $n_1$, the smaller the change of velocity distribution in the adsorption layer, and the greater the change of velocity distribution in the middle layer. The larger $\delta$ and $n_1$, the larger the deviation on log (film thickness) vs. log (speed) produced in the very thin film regime. In the region of the flow index ratio between 1.0 and 1.3, the difference in film thickness is significant. When the flow index of the adsorption layer is 1.6 times greater than the flow index of the middle layer, the adsorption layer is generally looked upon as a “solid-like”.

Keywords: TFEHL, Adsorption layer, Power law lubricants, Average viscosity.

1. INTRODUCTION

With the advances in manufacturing technology, modern machine elements can be designed and realized. Thin film lubrication (TFL) is one of the key issues as the film thickness is of several molecular lengths. It is crucial to understand the physics in such a microscopic scale. Many experimental results [1–4] show that the microscopic phenomena in TFL regime are substantially different from those in the elastohydrodynamic lubrication (EHL) regime. Therefore, new modeling techniques are needed for further research.

In TFL regime, the flow rheology should be considered for the novel characteristics of fluids. Guangteng and Spikes [1] found many common base fluids form layers of high viscosity on solid surfaces of about one to two molecular layers thickness by using an ultrathin film interferometry. These layers result in an enhancement of film thickness in rolling, concentrated contacts above that predicted from EHL theory based upon the bulk viscosity of the lubricant. Luo et al. [2] found that there is an ordered layer film between the dynamic film and the adsorbed film, which is more rigid than the dynamic film and less rigid than the adsorbed film, and is the transition lubrication from EHL to TFL and also the failure condition of liquid film. Hartl et al. [3] found that no fluid film remained in contact when motion was halted, and therefore it could be assumed that an immobile layer of aligned molecules adheres to both rubbing surfaces. Gee et al. [4] proposed that the properties of the fluid are essentially those of a Newtonian fluid as the film thickness is more than 30nm. As the film thickness is less than 30nm, the fluid exhibits obviously non-Newtonian characteristics. As described above, TFL has both the characteristics of fluid film and adsorption film, and the rheological behavior...
of the adsorption film is different from that of the fluid film. Hence, chu et al. [5] derived the modified Reynolds equation for thin film EHL with Newtonian fluid by means of the viscous adsorption theory. In addition, the thin film EHL analysis is performance on a surface forces model which includes van der waals and solvation forces. However, the rheological behavior for thin film lubrication exhibits non-Newtonian characteristics.

As described above, there is the phenomenon in the status of the thin film lubrication that the conventional tribology fails to explain. The measured film thickness in very thin film lubrication regime cannot be predicted by the traditional elastohydrodynamic lubrication theory. Therefore, there are several rheological models [6~11] have been developed which can be applied to thin film lubrication. However, so far the investigations about the non-Newtonian fluids in thin film lubrication are still quite limited. Non-Newtonian characteristics have been invariably observed in various lubrication problems. These may be due to the high shear rate and the high pressure gradient, or due to certain additives. In practice, almost all lubricants exhibit non-Newtonian behavior under certain circumstances. Moreover, the use of non-Newtonian fluids as lubricants has become increasingly important with the development of modern industrial materials. Various theories have been developed and analyzed to describe the flow behavior of non-Newtonian fluids. In engineering practice, lubricants are often modeled with the power law fluid which characterizes accurately both pseudoplastic and dilatant fluids—two important classes of non-Newtonian lubricants.

In this paper, a modified Reynolds equation for power-law fluids is derived for TFEHL by means of the viscous adsorption theory [16]. The lubricating film is modeled as three layers across the film: two adsorption layers near the solid surfaces and one middle layer. The viscosity and density of lubricant are varying with the pressure. It is known that the adsorbed layers may be formed from the environment on surfaces. As the film thickness is of the order of a few nanometers, these adsorbed layers will play an important role. The contact geometry of two rollers can be reduced to the contact geometry as a roller and a flat surface, as shown in Fig. 1. The fluid inertia and the body force are neglected as the fluid is thin. Hence, the steady-state momentum equations under pure rolling condition can be reduced to

\[ 0 \leq z \leq \delta : \]
\[ \frac{\partial p}{\partial x} = \frac{\partial}{\partial z} \left( m_1 \frac{\partial u_1}{\partial z} \right) \]
\[ \frac{\partial u_1}{\partial z} = \frac{\partial u_2}{\partial z} \]
\[ \delta \leq z \leq h/2 : \]
\[ \frac{\partial p}{\partial x} = \frac{\partial}{\partial z} \left( m_2 \frac{\partial u_2}{\partial z} \right) \]

Boundary conditions for Eqs. (2) and (3) are

(i) on solid surfaces, no slip conditions are utilized, i.e.
\[ z = 0, \quad u_1 = u_b \]

(ii) at the layer interface, velocity and shear stress matched conditions are utilized, i.e.
\[ z = \delta, \quad u_1 = u_2, \quad m_1 \frac{\partial u_1}{\partial z} = m_2 \frac{\partial u_2}{\partial z} \]

(iii) in the middle plane, symmetric conditions are utilized, i.e.
\[ z = h/2, \quad \frac{\partial u_2}{\partial z} = 0.0 \]

2. THEORETICAL ANALYSIS

The stress tensor and strain tensor for a power law fluid are related according to

\[ \tau = m_1 \frac{\partial u_1}{\partial z} \]

where \( m_1 \frac{\partial u_1}{\partial z} \) is usually called the equivalent viscosity of fluid. \( m \) is viscosity index and \( n \) is the flow index. The conditions of \( n > 1, n = 1, \) and \( n < 1 \) correspond to a dilatant fluid, Newtonian fluid, and pseudoplastic fluid, respectively.

2.1 Modified Reynolds Equation for TFEHL

Based on the viscous adsorption theory, the modified Reynolds equation for power-law fluids is derived for TFEHL. The lubricating film is modeled as three layers across the film: two adsorption layers near the solid surfaces and one middle layer. The viscosity and density of lubricant are varying with the pressure. It is known that the adsorbed layers may be formed from the environment on surfaces. As the film thickness is of the order of a few nanometers, these adsorbed layers will play an important role. The contact geometry of two rollers can be reduced to the contact geometry as a roller and a flat surface, as shown in Fig. 1. The fluid inertia and the body force are neglected as the fluid is thin. Hence, the steady-state momentum equations under pure rolling condition can be reduced to
With these boundary conditions, Eqs. (2) and (3) can be integrated directly to give the velocity components, i.e.

\[
0 \leq z \leq \delta:
\]

\[
u = \nu_0 + m_1 \left[ \left( \frac{n_1}{n_1 + 1} \right) - \left( \frac{dp}{dx} \right) \left( \frac{1}{m_1} \right) \right] 
\times \left[ \left( \frac{h}{2} \right)^{\frac{1-n_1}{m_1}} - \left( \frac{h - z}{2} \right)^{\frac{1-n_1}{m_1}} \right]
\] (5)

\[
\delta \leq z \leq h/2:
\]

\[
u = \nu_0 + m_1 \left[ \left( \frac{n_1}{n_1 + 1} \right) - \left( \frac{dp}{dx} \right) \left( \frac{1}{m_1} \right) \right] 
\times \left[ \left( \frac{h}{2} \right)^{\frac{1-n_1}{m_1}} - \left( \frac{h - \delta}{2} \right)^{\frac{1-n_1}{m_1}} \right]
\] (6)

Introducing these expressions into the continuity equation forms the modified Reynolds equation to give

\[
2\frac{\partial}{\partial x} \left[ \int_0^\delta \rho \nu_1 dz + \int_\delta^{h/2} \rho \nu_2 dz \right] = -\frac{\partial (\rho h)}{\partial t}
\] (7)

Therefore, the modified Reynolds equation can be written as:

\[
\frac{\partial}{\partial x} \left[ \rho m_1 \left( \frac{2n_1}{2n_1 + 1} \right) - \left( \frac{dp}{dx} \right) \left( \frac{1}{m_1} \right) \right] 
\times \left[ \left( \frac{h}{2} \right)^{\frac{1+n_1}{m_1}} - \left( \frac{h - \delta}{2} \right)^{\frac{1+n_1}{m_1}} \right]
\] (8)

Note that as \( \delta = 0.0 \), \( n_1 = n_2 = 1.0 \), and \( m_2 = m_1 \), Eq. (8) can be simplified into the classical Reynolds equation. The boundary conditions for Eq. (8) are

\[
p = 0 \text{, at } x = x_{\text{in}} \text{ (9a)}
\]

\[
p = \frac{dp}{dx} = 0 \text{, at } x = x_{\text{out}} \text{ (9b)}
\]

2.2 Film Thickness

Using the parabolic approximation for the geometry, as shown in Fig. 1, the lubricant film thickness is given as:

\[
h(x) = h_0 + \frac{x^2}{2R} + \delta(x)
\] (10)

where \( h_0 \) is a constant, \( \delta(x) \) is an elastic deformation. The elastic deformation can be written as:

\[
\delta(x) = -\frac{2}{\pi E} \int_{-\infty}^{\infty} \ln \left| \frac{x - x'}{x_0} \right| p(x') dx'
\] (11)

2.3 Lubricant Properties

The apparent viscosity and density are unknown and the thermal effect on the apparent viscosity and density is neglected. Dowson and Higginson [17] proposed the relationship between the density and pressure as:

\[
\rho = \rho_0 \left[ 1 + \frac{0.6 \times 10^{-9} p}{1 + 1.7 \times 10^{-9} p} \right]
\] (12)

The viscosity index-pressure relationship is the Roe-lands equation [18] as:

\[
m = m_0 \exp \left[ (9.67 \ln m_0) [-1 + (1 + 5.1 \times 10^{-9} p)^{1.710}] \right]
\] (13)

where \( m_0 \) is the viscosity at ambient pressure and \( z' \) is the pressure-viscosity index.

2.4 Force Balance Equation

The load balance equation for the line contact can be written as:

\[
\int_{-\infty}^{\infty} p(x) dx = w
\] (14)

where \( w \) is external load per unit width.

2.5 Average Viscosity

The average viscosity across the adsorption layer is defined as:

\[
m_{av} = \frac{1}{\delta} \int_0^{\delta} \left[ \frac{\partial h}{\partial x} \right]^{n+1} dx
\]

\[
m_{av} = \frac{1}{\delta} \int_0^{\delta} \left[ \frac{\partial h}{\partial x} \right]^{n-1} dx
\]

\[
m_{av} = \frac{1}{\delta} \left[ \frac{1}{m_2} \right]^{1-n} \left[ \frac{h}{2} \right]^{n} - \left( \frac{h - \delta}{2} \right)^{n+1}
\] (15)
The average viscosity across the middle layer is defined as:

\[ m_{m, av} = m_2 \times \frac{1}{(h/2 - \delta)^2} \int_{0}^{h/2 - \delta} \frac{du_2}{dz} \, dz \]

\[ = \frac{m_2}{(2m - 1)} \left( \frac{1}{m_2} \right)^{-1} \frac{\rho}{\rho_{m}} \left( \frac{h}{2 - \delta} \right)^{m_2 - 1} \]  

The average viscosity across the film is defined as:

\[ m_{f, av} = \frac{2\delta}{h} m_{av} + \frac{(h-2\delta)}{h} m_{m, av} \]  

3 RESULTS AND DISCUSSION

A typical problem with \( W = 4.55 \times 10^{-6}, U = 4.67 \times 10^{-13}, G = 2420, \) and \( S = 0.0 \) is solved and discussed. The dimensional parameters are \( w = 2000N, u_a = u_b = 0.005m/s, \delta = 0.5nm, m_1 = 0.041Pa - s^{n_1}, m_2 = 0.041Pa - s^{n_2}, \) and \( E' = 220GPa. \) Using a multilevel multi-integration (MLMI) algorithm, the solver developed by Hsu and Lee [19] is extended to obtain the film shape and pressure distribution under thin film elastohydrodynamic lubrication with non-Newtonian lubricants conditions. The coarsest auxiliary grid used in a full multigrid algorithm consists of \((12+1)\) nodes and the grid at level \( l \) includes \((12 \times 2^l + 1)\) nodes. Most of the cases presented in this paper use a domain extending from \( x_m = -4.0b \) to \( x_m = 1.4b. \) A grid size of 193 grids in the domain is used for the evaluation of pressure and elastic deformation. The properties of lubricants and rollers are shown in Tables 1 and 2, respectively.

When \( \delta = 0.0, \) the TFEHL model for power law fluids can be simplified into the traditional EHL model for power law fluids. Figure 2 shows the pressure distributions and the film shapes for lubricant modeled with one layer. The effect of flow indices \((n_1 = n_2 = 0.95, 1.00, 1.05)\) on the pressure distributions and the film shapes are discussed. The film thickness increases with the increase of the flow index. As the flow index increases, the dimple in the film shape moves towards the center \((X = 0.0)\) of contacts. The reason is that, when \( n \) increases the equivalent viscosity of lubricants rise with it. Under conditions of constant load, the force balance equation must be satisfied. The effect of the flow index produces an obvious difference in the pressure distribution. The greater the flow index, the smaller the pressure distribution in the Hertzian contact region, and the greater the pressure distribution in the inlet region.

As \( \delta = 0.0 \) and \( n_1 = n_2 = 1.0, \) the power law fluid model can be simplified to the Newtonian fluid model. As shown in Fig. 3, the pressure distributions and the film shapes are plotted for different flow indices \((n_1 = 1.0, 1.1, 1.2)\) with middle layer characterize Newtonian behavior.
fluid behavior \((n_2 = 1.0)\). The film thickness increases with the increase of the flow index of adsorber layer \((n_1)\). Furthermore, as the flow index \((n_1)\) becomes greater, the dimple in the film shape moves towards the center of the contacts. Therefore, we can obtain slightly smaller pressure distribution in the Hertzian contact region for greater flow index \((n_1)\). We can also obtain slightly larger pressure distribution outside the Hertzian contact region for greater flow index \((n_1)\) owing to the force balance condition.

In Fig. 4, the central film thickness versus the flow index ratio \((n_1/n_2)\) are plotted for various film thickness of the adsorption layer. The value of central film thickness increases as the flow index ratio increases. In the region of the flow index ratio between 1.0 and 1.3, the difference in central film thickness is significant.

When the flow index of the adsorber layer is 1.6 times greater than the flow index of the middle layer, the adsorption layer is generally looked upon as a “solid-like”. In the region of the flow index ratio greater than 1.6, the deviations of the central film thickness are almost 1.0nm for \(\delta = 0.5\)nm, and the deviations of the central film thickness are almost 2.0nm for \(\delta = 1.0\)nm. Spike [1] indicates that most of the fluids tested have thin layers of fluid on the two solid surfaces of enhanced viscosity compared to the bulk liquid and thus produce thicker-than-predicted lubricating films at low speeds. Therefore, the present model is reasonable to predict rheological characteristics for thin film lubrication.

Figure 5 shows the pressure distributions and film shapes for four different cases of flow rheology, i.e. \((m_1, m_2, n_1, n_2)\) = (case A: 0.041, 0.041, 1.0, 1.0) = EHL, (case B: 0.041, 0.041, 1.1, 1.0), (case C: 0.041, 0.041, 1.1, 1.1), and (case D: 0.41, 0.041, 1.0, 1.0), respectively. With the same adsorption thickness \((\delta = 0.5)\)nm), viscosity of middle layer \((m_2 = 0.041)\) and flow index \((n_1 = n_2 = 1.0)\), the film thickness increases with the increase of the viscosity of adsorption layer. The discrepancy of the central film thickness between \(m_1 = 0.41\) and \(m_1 = 0.041\) is almost 15%. The difference in pressure distribution between \(m_1 = 0.41\) and \(m_1 = 0.041\) is very small. With the same \(\delta, m_1, m_2\) and \(n_2\), the film thickness increases with the increase of \(n_1\). The discrepancy of the central film thickness between \(n_1 = 1.1\) and \(n_1 = 1.0\) is about 12%. The difference in pressure distribution between \(n_1 = 1.1\) and \(n_1 = 1.0\) is very small, except in the outlet region. With the same \(\delta, m_1, m_2\) and \(n_1\), the film thickness increases with the increase of \(n_2\). The discrepancy of the central film thickness between \(n_2 = 1.1\) and \(n_2 = 1.0\) is increased to about 130%. The pressure distribution of \(n_2 = 1.1\) is smaller than that of \(n_2 = 1.0\) in the Hertzian contact region, and the pressure distribution of \(n_2 = 1.1\) is larger than that of \(n_2 = 1.0\) outside the Hertzian contact region. Due to the ratio of the middle layer thickness to the total film thickness is large, the effect of \(n_2\) is greater than the effect of \(n_1\) on the thin film thickness. The effect of viscosity index of adsorption layer \((m_1)\) on the thin film thickness is greater than the effect of \(n_1\) on the thin film thickness.

According to Eqs. (5) and (6), velocity distribution can be obtained. Figures 6(a) and 6(b) show the velocity distribution across the film in the X-direction at the vicinity of \(X = -0.09\) and \(X = 0.6125\) for various flow indices. The velocity and the shear stress must be the same at the interface of the adsorption layer and the middle layer. Furthermore, the velocity distribution is influenced by the film thickness, the viscosity, the flow index, and pressure gradient. It also is evident that the larger the viscosity, the less the velocity varies. Therefore, the change of velocity distribution in the adsorption layer is smaller than that in the middle layer. The greater the flow index of the adsorption layer \((n_1)\), the smaller the change of velocity distribution in the adsorption layer, and the greater the change of velocity distribution in the middle layer. Therefore, it is evident that the flow indices of the adsorption layer
influences significantly the lubrication characteristics of the contact conjunction for the present model. Furthermore, the present model can also reasonable predict the velocity of the adsorption layer affected by surface energy in course of abrasive shear.

According to the experiments performed by Guangteng and Spike [1], all fluids except hexadecane and the cyclic hydrocarbon traction fluid exhibit a deviation from linearity on a log (film thickness) vs. log (speed) in the very thin film regime. This deviation produces a thicker film than predicted from the EHL theory in all cases. It is therefore concluded that the viscosity of the adsorption layer is greater than that of the middle layer. This is due to the structural change takes place in the hydraulic molecules subject to the reaction of the surface energy in the course of abrasive shear which forces those molecules to be arranged in sequence on the upper and lower surfaces, respectively. Figure 7 shows that the central film thickness is less than 30nm, the central film thickness with speed variations using present model deviates from that predicted by the classical EHD theory for \( n_1 = 1.1 \) and \( n_1 = 1.2 \). For the absorption layer of 0.5nm and the speed of 0.001m/s, the deviations are 0.59nm for \( n_1 = 1.1 \), 0.81nm for \( n_1 = 1.2 \), respectively. For the absorption layer of 1.0nm and the speed of 0.001m/s, the deviations are 1.11nm for \( n_1 = 1.1 \), 1.54nm for \( n_1 = 1.2 \), respectively. The larger the flow index, the larger the deviation on log (film thickness) vs. log (speed) produced in the very thin film regime. Furthermore, it is found that the deviation in the very thin film regime strongly depends upon the flow indices and thickness of the adsorption layer, i.e., the larger the thickness and flow index of the adsorption layer, the larger the deviation produced in the very thin film regime. It is evident that the present model for \( n_1 > n_2 \) is reasonable to predict the deviations and turning point in the very thin film regime.

To investigate the effect of film thickness on the average viscosity, the average viscosity versus film thickness are plotted in Fig. 8. The average viscosity across the film is defined in Eq. (17). It is seen from this definition that the weighting ratio of the adsorption layer increases with the decrease in film thickness. Hence, the less the film thickness is, the larger the average viscosity becomes. For the film thickness of around 6.10nm, \( \delta = 0.5 \)nm, and \( n_1 = 1.1 \), the average viscosity calculated using the present model is 1.65 times larger than that obtained from the classical EHL model. For the film thickness of around 6.13nm, \( \delta = 0.5 \)nm, and \( n_1 = 1.2 \), the average viscosity calculated using the present model is 2.65 times larger than that obtained from the classical EHL model. Therefore, the larger the flow index of the adsorption layer, the larger the average viscosity produced in the very thin film regime. Guangteng and Spike [1] indicate that the viscosity of di-(2-ethylhexyl) sebacate rises by about a factor of five within 2nm of the solid surface. Hartl et al. [3] indicates that the presence of additives results in the formation of boundary films with seven times the viscosity of the base oil. Therefore, the present model is reasonable to predict the average viscosity in TFL.

Figure 9 shows the film thickness varying with the load. The minimum film thickness calculated using the present model is larger than that calculated using the traditional EHL model because the average viscosity in TFL is prominently larger than that in bulk. It can be seen that the minimum film thickness rises with the increase in the flow index. The reason is that, when \( n_1 \) increases the average viscosity of lubricants rises with it. The minimum film thickness decreases slightly with an increase of load, but the minimum film thickness increases more obviously with a slight decrease of the load.

4. CONCLUSIONS

A layered-rheological model for TFEHL is proposed. A modified Reynolds equation for power law fluid is
Fig. 7 Log (film thickness) vs. log (speed) using present model with three different lubricants ($w = 2000 N$, $z = 0.3329$, and $S = 0.0$)

Fig. 8 Variation of average viscosity versus film thickness using present model with three different lubricants ($w = 2000 N$, $u_a = u_b = 0.005 m/s$, and $z = 0.3329$)

derived by utilizing the viscous adsorption theory. The results can be summarized as follows:

1. The film thickness increases with the increase of the flow index of adsorption layer ($n_1$), but the effect of the $n_1$ produces a very small difference in the pressure distributions.

2. The value of central film thickness increases as the flow index ratio increases. In the region of the flow index ratio between 1.0 and 1.3, the difference in central film thickness is significant. When the flow index of the adsorption layer is 1.6 times greater than the flow index of the middle layer, the adsorption layer is generally looked upon as a “solid-like”.

3. The film thickness affected by viscosity index of adsorption layer ($m_1$) is greater than that affected by $n_1$.

4. The greater the $n_1$, the smaller the change of velocity distribution in the adsorption layer, and the greater the change of velocity distribution in the middle layer.

5. The larger the thickness and flow index of the adsorption layer, the larger the deviation on log (film thickness) vs. log (speed) produced in the thin film regime.

6. In thin film region, the average viscosity predicted by the present model is better than that predicted by the traditional EHD theory.

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**NOMENCLATURE**

- $b$: semiwidth of hertzian contact (m)
- $E'$: equivalent Young's modulus (Pa)
- $G$: dimensionless material parameter, $\alpha E'$
- $h$: film thickness (m)
- $h_c$: central film thickness
- $h_0$: constant defined in Eq. (10)
- $H$: dimensionless film thickness, $hR/b^2$
- $Hc$: dimensionless central film thickness
- $m_1$: viscosity index for adsorption layer lubricant $(Pa - s^{n_1})$
- $m_2$: viscosity index for middle layer lubricant $(Pa - s^{n_2})$
- $\bar{m}$: dimensionless viscosity, $m/m_0$
REFERENCES