Dynamics of liquid meniscus bridge of a vibrating disk: consideration of flow rheology

W.-L. Li

Institute of Nanotechnology and Microsystems Engineering, National Cheng Kung University, No. 1, University Road, Tainan 701, Taiwan, Republic of China
E-mail: dragon@mail.mina.ncku.edu.tw

Abstract: A modified Reynolds equation, which includes the effect of flow rheology, is derived to describe the flow behaviour of lubricant between the space of a magnetic head slider and a disk. Under the assumptions of a small vibration of the spacing, and zero contact angle of the liquid–solid interface, the dynamics of a liquid meniscus disk of finite radius is analysed. The time-dependent modified Reynolds equation is linearised, and solved, under the boundary condition considering Laplace pressure. The results show that the pressure and load carrying capacity consist of three terms, that is, the static meniscus force term, the spring term by the dynamic Laplace pressure, and time-dependent damping term by the flow rheology of the fluid. The flow rheology affects the static meniscus forces and the damping forces significantly as compared to the spring forces. The effects of flow rheology on the load carrying capacity are also discussed.

1 Introduction

To promote the performance of a hard disk drive (HDD), there are basically two contrary strategies: either increasing the recording density by reducing the flying height of magnetic slider, or decreasing the access time by increasing the disk velocity (thus, increasing the flying height) for gas-lubricated slider bearings. The desire to increase the magnetic recording densities and to decrease the access time motivates the study of liquid-lubricated slider bearings. The reduction of flying height can be achieved significantly by introducing the non-Newtonian shear thinning lubricant at high shear rate. In addition, the liquid-lubricated slider bearing is much stiffer than a conventional air bearing, and thus permits the slider to approach the disk surface more closely without actually contacting it [1, 2]. Many apparatus are developed to realise this concept of liquid-lubricated slider bearings [3–6].

In addition, the dynamics of liquid meniscus bridge of intermittent contact slider is considered [7]. The dynamic characteristics of a meniscus bridge between solid surfaces are performed. Newtonian fluid is considered as the lubricant. In practice, the flow rheology and dynamic characteristics of lubricant are very important for the designing on the novel head disk interface (HDI) or recording system. The measurement of the linear viscoelastic properties of small volumes of a fluid is also developed by modelling the squeeze flow [8] in micro-rheometers. The perfluoropolyether (PFPE), that is used as a lubricant on a magnetic disk, formed between the diamond-like carbon (DLC) surface of a magnetic disk and a diamond probe tip is also considered to study the dynamic coefficients (spring and damping coefficients) of the lubricant bridge [9]. The lubes on the disk can be modelled as non-Newtonian fluids due to their shear thinning effects.

In this study, a modified Reynolds equation is derived to include the effect of flow rheology. The time-dependent modified Reynolds equation is linearised under the conditions of small variation of the spacing, and is solved analytically under the boundary condition considering Laplace pressure.

2 Derivation of modified Reynolds equation

The flow rheology is considered by introducing the non-Newtonian power-law lubricant in the analysis of a finite meniscus ring as shown in Fig. 1. The usual assumptions
of thin film lubrication [10], constitutive law of power law fluid, zero contact angle of the liquid–solid interface and smooth solid surfaces are made in the present derivation. The lower surface is assumed to be of no vibration, and the vibration of the upper surface is considered as a disturbance from undulation of a disk surface. The flow rheology (stress–strain rate relation) is \( \tau = m |\partial u/\partial z|^{n-1}(\partial u/\partial z) \), where \( n \) is the flow index of the power-law fluid and \( m \) is a viscosity constant. Using the hydrodynamic lubrication assumptions applicable to thin films, the approximate equation in the \( r \)-direction for the flow is [10]

\[
-\frac{\partial p}{\partial r} + \frac{\partial}{\partial z} \left[ m \frac{\partial u}{\partial z} \right] = 0
\]

where \( p \) is the pressure. The equation of continuity is

\[
\frac{1}{r} \frac{\partial (ru)}{\partial r} + \frac{\partial u}{\partial z} = 0
\]

The no-slip boundary conditions at the solid interfaces are

\[
u = 0, \quad z = 0, \quad \text{and} \quad u = 0, \quad \text{at} \quad z = b
\]

The momentum (1) gives

\[
u = \frac{1}{m} \frac{d \rho}{d t} \left| \frac{1}{m} \frac{d \rho}{d t} \right|^{(1-n)/n} \frac{n}{n+1} \left[ \frac{b}{2} - \frac{\delta(t)}{2} \right]^{(n+1)/n} - \frac{\delta(t)}{2}^{(n+1)/n}
\]

Substituting into continuity (2), we have

\[
\frac{1}{r} \frac{d}{d r} \int_0^b u r d z = -\frac{\partial h}{\partial t}
\]

The integral

\[
\int_0^b u d z = \int_0^{b/2} u d z + \int_{b/2}^b u d z
\]

\[
= \frac{1}{m} \frac{d \rho}{d t} \frac{1}{m} \frac{d \rho}{d t} \left| \frac{1}{m} \frac{d \rho}{d t} \right|^{(1-n)/n} - \frac{2n}{2n+1} \left( \frac{b}{2} \right)^{(2n+1)/n}
\]

Therefore, we have the modified Reynolds equation for non-Newtonian power-law lubricant, that is

\[
\frac{1}{r} \frac{d}{d r} \left[ \frac{1}{m} \frac{d \rho}{d t} \right] \left[ \frac{1}{m} \frac{d \rho}{d t} \right]^{(1-n)/n} - \frac{2n}{2n+1} \left( \frac{b}{2} \right)^{(2n+1)/n} = \frac{\partial h}{\partial t}
\]

The boundary conditions are

(1) at \( r = 0, \) \( dp/dr = 0, \) and

(2) at \( r = r_{10} \), \( p = \gamma ((-2/b) + (1/r_1)) \approx \gamma [(-2/b_0)(1 - \delta(t)) + (1/r_{10})(1 + (1/2)\delta(t))] \)

Here, we have \( b = b_0(1 + \delta(t)), \) and \( r_1^2 \approx r_{10}^2(1 - \delta(t)) \) from the mass conservation of meniscus liquid.

Then, the pressure distribution is solved, that is

\[
p = \gamma \left( \frac{2}{b_0} - \frac{1}{r_{10}} \right) + \gamma \left( \frac{2}{b_0} + \frac{1}{2r_{10}} \right) \delta(t)
\]

\[+ A \frac{1}{r_{10}^{n+1}} \frac{n}{n+1} \frac{db}{dr} \frac{d}{dr} \left( \frac{b}{2} \right)^{-n+1} \]

where \( A = (2n/2n + 1)(1/m)^{1/n}(1/2)^{(2n+1)/n} \).

The load carrying capacity is obtained by integrating the pressure distribution

\[
F(t) = \int_0^{r_{10}(t)} 2\pi r \cdot p(r, t) dr
\]

\[= -2\pi r_{10}^2 \gamma \left( \frac{1}{b_0} - \frac{1}{2r_{10}} \right) + 4\pi r_{10}^2 \gamma \left( \frac{1}{b_0} - \frac{1}{8r_{10}} \right) \delta(t)
\]

\[+ 2\pi A \frac{1}{r_{10}^{n+1}} \frac{n}{n+1} \frac{1}{n+3} \left( \frac{1}{n+3} - \frac{1}{2} \right)
\]

\[\times r_{10}^{n+1} \frac{db}{dr} \frac{d}{dr} \left( \frac{b}{2} \right)^{-n+1} \delta(t)
\]

The three terms on the RHS of (9) are the static meniscus force term, \( F_3(t) \), spring term by the dynamic Laplace pressure, \( F_2(t) \), and time-dependent damping term by the flow rheology of the fluid, \( F_1(t) \). Only the damping term is a function of the flow index \( (n) \). The effects of flow rheology and surface tension on the load carrying capacities will be discussed later. For comparison of [7] with the present results, the special case of Newtonian fluid, that is, \( n = 1 \), is considered. We have \( A = 1/12 m \), and the load carrying capacity becomes

\[
F(t) = -2\pi r_{10}^2 \gamma \left( \frac{1}{b_0} - \frac{1}{2r_{10}} \right) + 4\pi r_{10}^2 \gamma \left( \frac{1}{b_0} - \frac{1}{8r_{10}} \right)
\]

\[\times \delta(t) - \frac{3\pi r_{10}^2}{2 b_0} \delta(t)
\]

that is the same as that proposed in [7].
3 Results and discussion

The discussion parameters are: \( f = 100 \text{ Hz} \), \( r_{10} = 5 \text{ mm} \), \( \delta(t) = 10^{-3} \sin(2 \pi f t) \), \( m = 10^{-3} \text{ Pa} \cdot \text{s}^2 \), and \( h_0 = 5 \mu \text{m} \). The effects of flow rheology on the load carrying capacity are plotted as shown in Fig. 2. The results show that the effect of flow index \( (n) \) on the load carrying capacity is significant on the damping term, \( F_3(t) \). The static meniscus force term, \( F_1(t) \), and the spring term \( F_2(t) \), are functions of surface tension \( (\gamma) \), and are independent of the flow index \( (n) \). From Fig. 2, the static meniscus force term, \( F_1(t) \), is of the main contributions on the load carrying capacities as compared to the other two terms. The amplitude of the spring term, \( F_2(t) \), is negligible as compared to the damping term, \( F_3(t) \). Therefore, the static meniscus force term affects the level of the load carrying capacity, and the damping term affects the amplitude of the load carrying capacity. The special case of \( n = 1 \) is the same as that in [7]. The shear thinning effects show that the amplitude of the load carrying capacity decreases as the flow index decreases.

In Fig. 3, the effects of surface tension on \( F(T) \) are discussed. The load carrying capacity, \( F(t) \), is plotted as functions of time for various surface tensions \( (\gamma) \). The results show that the surface tension affects \( F(t) \) significantly. Low surface tension leads to low Laplace pressure, and results in low attractive forces. From Figs. 2 and 3, we can observe that the resultant load carrying capacity is always in the negative regions. It means that the attractive force is acting in the meniscus ring. The static meniscus force dominates in (9) because of the small vibration amplitude, \( \delta(T) \). Therefore, the larger the surface tension, the larger the attractive force.

4 Conclusion

In this paper, the effects of surface tension as well as the flow rheology on the performance of bearing are discussed analytically. The modified Reynolds equation is derived to include the effect of flow rheology. The Laplace pressure
appears at the meniscus interface as a dominant factor that affects the attractive force significantly, and the flow rheology affects the force amplitude. The present model has the potential application on the modelling of HDI for novel sliders.

5 Acknowledgment

This research was supported by NSC at Taiwan, Contract No. NSC 96-2221-E-006-337.

6 References


doi: 10.1049/mnl:20090003 © The Institution of Engineering and Technology 2009