The partially wetted bearing—extended Reynolds equation

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Received 11 May 2005; received in revised form 1 January 2006; accepted 9 January 2006
Available online 28 February 2006

Abstract

The artificial no-slip boundary conditions on the liquid/solid interfaces are traditionally used widely. Due to the advances on the measurement technique and interface sciences, the applications of no-slip boundary conditions on micro-systems are challenged continuously. The ‘slip effects’ are observed in small clearance measurement or by treating the surfaces hydrophobic. The non-Newtonian power-law fluid as well as the Navier-slip boundary conditions is considered in the partially wetted bearings. A perturbation technique is utilized to derive the extended Reynolds equations. The analysis applied either to Couette–dominated highly non-Newtonian fluids, or to Newtonian fluids with arbitrary Couette-Poiseuille components. Finally, the effects of slip parameters on the bearing performances are discussed.

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Keywords: Non-Newtonian fluid; Navier slip; Power-law fluid

1. Introduction

Due to the advances on the measurement technique and interface sciences, the applicability of no-slip boundary conditions is challenged. The experiments observed that the slip occurs on smooth surfaces due to the weak ‘link’ between the fluid and solid surfaces [1–5]. In addition, the velocity slips are observed in small clearance measurement as well as by treating the surfaces hydrophobic. Detail reviews about slip phenomena are shown in the article by Granick et al. [6].

The studies of the slip effects on simple Newtonian fluid come from computer simulations [7–12], and experiments [2–4,13–20]. The surface force apparatus (SFA) [3,4,16,17] and atomic force microscope (AFM) [15,18] are used to measure the deviations of the squeeze forces for a micro-fluid as compared to that for a Newtonian fluids with no-slip boundary conditions. In addition, the relations between the velocity slips and mass flow rate are discussed.

Vinogradova [21] have studied the relations between the hydrodynamic forces and squeeze velocities of SFA analytically. The effects of Navier-slip conditions ($\eta v_s \equiv \sigma_s$, where $v_s$ is the slip velocity, $\sigma_s$ the shear stress at the surface, $\eta$ the viscosity and $b$ the slip length) on the hydrodynamic pressure are also considered in her derivation. The slip correction factors are derived. The slip length ($b$) depends on the kind of material, fluids, the contact angles or wettability between liquid and solid interface, and flow velocities [6]. Some slip lengths measurements obtained on wetting and non-wetting surface by different research groups are tabled and discussed [22]. Typical slip lengths as large as submicron [22] to micron [16] are obtained. Many surface chemists study the above problems.

In the application on lubrication problems, Spikes derived the extended Reynolds equation for half-wetted bearings [23,24]. No-slip conditions are imposed on the moving surface, and the lubricant can slip against the stationary bearing surface at a critical shear stress. As a critical surface shear stress is reached, and once slip begins, it takes place at a constant slip length [25]. The results show that the ‘half-wetted’ bearing is able to combine good load...
support resulting from fluid entrainment with very low friction due to very low or zero Couette friction. Some non-Newtonian properties occur at such a small clearance [3,15], especially for micro-bearings with non wetted bearing surfaces.

In this paper, the non-Newtonian power-law fluid [26] as well as the Navier-slip boundary conditions is considered. A perturbation technique is utilized to derive the extended Reynolds equations. The analysis applied either to Couette-dominated highly non-Newtonian fluids, or to Newtonian fluids with arbitrary Couette–Poiseuille components. Finally, the effects of slip parameters on the bearing performances are discussed. The present model has potential application on micro-bearings with partially wetted bearing surfaces.

2. Derivation

The derivations of the extended Reynolds-type equation and velocity profiles are quite similar to that by Dien and Elrod [26]. The Navier slips are assumed on the lubricating boundaries. Under the usual assumption of lubrication formulation [26], the momentum equations are

\[ \frac{\partial}{\partial z} \left( \mu \frac{\partial u}{\partial z} \right) = \frac{\partial p}{\partial x}, \]  
\[ \frac{\partial}{\partial z} \left( \mu \frac{\partial v}{\partial z} \right) = \frac{\partial p}{\partial y}. \]  

The viscosity, \( \mu = \mu(I) \), is dependent on the second invariant \( I \) of the strain rate tensor, i.e.

\[ I = \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2. \]  

We assume that the strain rates within the fluid are principally generated by the relative surface velocities. Thus, the analysis applies either to Couette–dominated highly non-Newtonian fluids, or to Newtonian fluids with arbitrary Couette–Poiseuille components. Therefore, the pressure gradient can be expressed as

\[ \nabla p = e \nabla \pi \]  
and the velocities are

\[ u = u_0(x,y) + \epsilon u_1(x,y) + \cdots, \]
\[ v = v_0(x,y) + \epsilon v_1(x,y) + \cdots. \]

Then, the second invariant can be expressed as

\[ I = \left( \frac{\partial u_0}{\partial z} \right)^2 + \left( \frac{\partial v_0}{\partial z} \right)^2 + \left( \frac{\partial u_1}{\partial z} \right)^2 + \cdots \]
\[ + \epsilon^2 \left\{ \frac{\partial^2 u_0}{\partial z^2} \frac{\partial u_1}{\partial z} + \frac{\partial u_0}{\partial z} \frac{\partial^2 v_1}{\partial z^2} \right\} + \cdots \]
\[ = I_0 + \epsilon I_1 + \cdots, \]  

where

\[ I_0 = \left( \frac{\partial u_0}{\partial z} \right)^2 + \left( \frac{\partial v_0}{\partial z} \right)^2, \quad I_1 = 2 \left\{ \frac{\partial u_0}{\partial z} \frac{\partial u_1}{\partial z} + \frac{\partial u_0}{\partial z} \frac{\partial v_1}{\partial z} \right\}. \]
The viscosity can be expressed as
\[ \mu = \mu(I_0 + eI_1 + ...) = \mu(I_0) + e \frac{\partial \mu}{\partial I} |_{I=I_0} I_1 + ... \]
= \mu_0 + e\mu_1, \quad (9)
where
\[ \mu_0 = \mu(I_0), \quad \mu_1 = \frac{\partial \mu}{\partial I} |_{I=I_0} I_1. \quad (10) \]
Substitute Eqs. (5)-(10) into Eqs. (1) and (2), we have
\[ \frac{\partial}{\partial z} \left[ (\mu_0 + e\mu_1) \frac{\partial u}{\partial z} (u_0 + eu) \right] = \frac{\partial p}{\partial x}, \]
\[ \frac{\partial}{\partial z} \left[ (\mu_0 + e\mu_1) \frac{\partial v}{\partial z} (v_0 + ev) \right] = \frac{\partial p}{\partial y}. \quad (11) \]
Therefore, the zero-order equations are
\[ \frac{\partial}{\partial z} \left[ \mu_0 \frac{\partial u_0}{\partial z} \right] = 0, \quad \frac{\partial}{\partial z} \left[ \mu_0 \frac{\partial v_0}{\partial z} \right] = 0. \quad (12) \]
The first-order equations are
\[ \frac{\partial}{\partial z} \left( \mu_0 \frac{\partial u_1}{\partial z} + \mu_1 \frac{\partial u_0}{\partial z} \right) = \frac{\partial p}{\partial x}, \]
\[ \frac{\partial}{\partial z} \left( \mu_0 \frac{\partial v_1}{\partial z} + \mu_1 \frac{\partial v_0}{\partial z} \right) = \frac{\partial p}{\partial y}. \quad (13) \]
The related boundary conditions are
at \( z = 0 \), \( u_0 = u_{1x} + b_1 \frac{\partial u_0}{\partial z} \),
\[ v_0 = v_{1y} + b_1 \frac{\partial v_0}{\partial z}, \quad u_1 = b_1 \frac{\partial u_1}{\partial z}, \quad v_1 = b_1 \frac{\partial v_1}{\partial z}. \quad (14) \]
at \( z = h \), \( u_0 = u_{2x} - b_2 \frac{\partial u_0}{\partial z} \),
\[ v_0 = v_{2y} - b_2 \frac{\partial v_0}{\partial z}, \quad u_1 = -b_2 \frac{\partial u_1}{\partial z}, \quad v_1 = -b_2 \frac{\partial v_1}{\partial z}, \quad (15) \]
where \((u_{1x}, v_{1y})\) and \((u_{2x}, v_{2y})\) are boundary velocities.
Integrating Eq. (12) w.r.t. \( z \), we have
\[ \mu_0 \frac{\partial u_0}{\partial z} = \tau_{xz} = \text{constant}, \quad \mu_0 \frac{\partial v_0}{\partial z} = \tau_{yz} = \text{constant}. \quad (16) \]
From
\[ \mu_0^2 \left( \left( \frac{\partial u_0}{\partial z} \right)^2 + \left( \frac{\partial v_0}{\partial z} \right)^2 \right) = \mu_0^2 I_0 = \tau_{xz}^2 + \tau_{yz}^2 = \text{constant}. \quad (17) \]
\( \mu_0 \) and \( I_0 \) can be regarded as constants (\( \mu_0 = \mu(I_0) \)).
Integrating Eq. (16) w.r.t. \( z \), we have
\[ u_0 = \frac{1}{\mu_0} \tau_{xz} z + c_1, \quad v_0 = \frac{1}{\mu_0} \tau_{yz} z + c_2 \quad (18) \]
with \( c_1 \) and \( c_2 \) to be determined.

Therefore, the velocity components are
\[ u_0 = u_{1x} + \frac{z + b_1}{h + b_1 + b_2} (u_{2x} - u_{1x}), \]
\[ v_0 = v_{1y} + \frac{z + b_1}{h + b_1 + b_2} (v_{2y} - v_{1y}), \quad (19) \]
\[ u_1 = D_x \left[ \frac{z^2}{2} - \frac{(z + b_1) h + h + b_2}{2 (h + b_1 + b_2)} \right], \]
\[ v_1 = D_y \left[ \frac{z^2}{2} - \frac{(z + b_1) h + h + b_2}{2 (h + b_1 + b_2)} \right], \quad (20) \]
where
\[ \begin{align*}
D_x &= \frac{1}{\mu_0} \frac{\partial p}{\partial x} - \frac{V_x}{h + b_1 + b_2} \\
&\times \left( \frac{V_x \frac{\partial p}{\partial x} + V_y \frac{\partial p}{\partial y}}{h + b_1 + b_2} \right) \\
&\times \left( \frac{2 \frac{\partial p}{\partial y} |_{I=I_0}}{\mu_0^2 b} \right) \\
D_y &= \frac{1}{\mu_0} \frac{\partial p}{\partial y} - \frac{V_y}{h + b_1 + b_2} \\
&\times \left( \frac{V_y \frac{\partial p}{\partial x} + V_y \frac{\partial p}{\partial y}}{h + b_1 + b_2} \right) \\
&\times \left( \frac{2 \frac{\partial p}{\partial y} |_{I=I_0}}{\mu_0^2 b} \right)
\end{align*} \quad (21) \]
and \( V_x = u_{2x} - u_{1x} \), \( V_y = v_{2y} - v_{1y} \).
Substituting \( u_0 \) and \( u_1 \) into Eqs. (5) and (6), we can obtain the velocity profiles, i.e.
\[ u = u_{1x} + \frac{z + b_1}{h + b_1 + b_2} (u_{2x} - u_{1x}) \\
+ \frac{b_1}{2} \frac{\partial D_x}{\partial z} \left[ \frac{z^2}{2} - \frac{(h + b_2)(z + b_1)}{2 (h + b_1 + b_2)} \right], \]
\[ v = v_{1y} + \frac{z + b_1}{h + b_1 + b_2} (v_{2y} - v_{1y}) \\
+ \frac{b_1}{2} \frac{\partial D_y}{\partial z} \left[ \frac{z^2}{2} - \frac{(h + b_2)(z + b_1)}{2 (h + b_1 + b_2)} \right]. \quad (22) \]
With the following relations for power-law fluid
\[ \mu = m l (n - 1)/2. \quad (23) \]
The mass flow rates are
\[ \dot{m}_x = \sqrt{\rho \nu u_0 dz} \]
\[ = \rho u_{1x} h + \rho h \frac{b_1 + b_2}{2} V_x - \rho \frac{h^3}{12 \mu_0} \]
\[ \times \left[ 1 + \frac{3 (b_1 + b_2)}{h} - \frac{3 (b_1 - b_2)^2}{2 (h + b_1 + b_2)} \right] \\
\times \left( \frac{\frac{\partial p}{\partial x} - s_x (s \bullet \nabla p) \frac{n - 1}{n}}{n} \right). \]
\[
\dot{m}_y = \int_0^h \rho v \, dz
\]
\[
= \rho v_1h + \frac{\rho h}{2} \frac{h + 2b_1}{h + b_1 + b_2} V_y
\]
\[
- \frac{\rho h}{12\mu_0} \left[ \frac{1 + 3}{h} \frac{b_1 + b_2}{h} - \frac{3(b_1 - b_2)^2}{h(h + b_1 + b_2)} \right]
\]
\[
\times \left[ \frac{\partial p}{\partial y} - s_y (s \cdot \nabla p) \frac{n - 1}{n} \right],
\]
(24)
\[
\text{where}
\]
\[
s = s_x i + s_y j, \quad s_x = \frac{V_x}{\sqrt{V_x^2 + V_y^2}}, \quad s_y = \frac{V_y}{\sqrt{V_x^2 + V_y^2}}.
\]

The extended Reynolds equation can be obtained from
\[
\frac{\partial}{\partial x} (m_i \dot{v}_i) + \frac{\partial}{\partial y} (m_j \dot{v}_j) = -\frac{\partial}{\partial t} (p h).
\]
(25)

Thus, we have
\[
\frac{\partial}{\partial x} \left\{ h^3 (h + b_1 + b_2)^{n-1} \left[ 1 + 3 \frac{b_1 + b_2}{h} - \frac{3(b_1 - b_2)^2}{h(h + b_1 + b_2)} \right] \right\}
\]
\[
\times \left[ \left( 1 - s_x \frac{n - 1}{n} \right) \frac{\partial p}{\partial x} - \frac{n - 1}{n} s_x s_y \frac{\partial p}{\partial y} \right] \]
\[
+ \frac{\partial}{\partial y} \left\{ h^3 (h + b_1 + b_2)^{n-1} \right\}
\]
\[
\times \left[ 1 + 3 \frac{b_1 + b_2}{h} - \frac{3(b_1 - b_2)^2}{h(h + b_1 + b_2)} \right]
\]
\[
\times \left[ \left( 1 - s_y \frac{n - 1}{n} \right) \frac{\partial p}{\partial y} + \left( 1 - s_y \frac{n - 1}{n} \right) \frac{\partial p}{\partial y} \right] \]
\[
= 12m \left( V_x^2 + V_y^2 \right)^{1/2} \frac{\partial}{\partial x} \left( V_y h \frac{h + 2b_1}{2 h + b_1 + b_2} \right)
\]
\[
+ 12m \left( V_x^2 + V_y^2 \right)^{1/2} \frac{\partial}{\partial y} \left( V_x h \frac{h + 2b_1}{2 h + b_1 + b_2} \right)
\]
\[
+ 12m \left( V_x^2 + V_y^2 \right)^{1/2} \frac{\partial h}{\partial t},
\]
(26)

If \( s_x = 1, s_y = 0 \), then we have
\[
\frac{\partial}{\partial x} \left\{ h^3 (h + b_1 + b_2)^{n-1} \left[ 1 + 3 \frac{b_1 + b_2}{h} - \frac{3(b_1 - b_2)^2}{h(h + b_1 + b_2)} \right] \right\}
\]
\[
\times \left[ \left( 1 - s_x \frac{n - 1}{n} \right) \frac{\partial p}{\partial x} - \frac{n - 1}{n} s_x s_y \frac{\partial p}{\partial y} \right] \]
\[
+ \frac{\partial}{\partial y} \left\{ h^3 (h + b_1 + b_2)^{n-1} \right\}
\]
\[
\times \left[ 1 + 3 \frac{b_1 + b_2}{h} - \frac{3(b_1 - b_2)^2}{h(h + b_1 + b_2)} \right]
\]
\[
\times \left[ \left( 1 - s_y \frac{n - 1}{n} \right) \frac{\partial p}{\partial y} \right] \]
\[
= 12m \left( V_x^2 + V_y^2 \right)^{1/2} \frac{\partial}{\partial x} \left( V_y h \frac{h + 2b_1}{2 h + b_1 + b_2} \right)
\]
\[
+ 12m \left( V_x^2 + V_y^2 \right)^{1/2} \frac{\partial h}{\partial t},
\]
(27)

If \( s_x = 1, s_y = 0, \) and \( n = 1 \), then we have
\[
\frac{\partial}{\partial x} \left\{ h^3 \left[ 1 + 3 \frac{b_1 + b_2}{h} - \frac{3(b_1 - b_2)^2}{h(h + b_1 + b_2)} \right] \right\}
\]
\[
+ \frac{\partial}{\partial y} \left\{ h^3 \left[ 1 + 3 \frac{b_1 + b_2}{h} - \frac{3(b_1 - b_2)^2}{h(h + b_1 + b_2)} \right] \right\}
\]
\[
= 12m \frac{\partial}{\partial x} \left( V_y h \frac{h + 2b_1}{2 h + b_1 + b_2} \right) + 12m \frac{\partial h}{\partial t},
\]
(28)

If \( s_x = 1, s_y = 0, \) and \( b_1 = b_2 = 0, \) then we have
\[
\frac{\partial}{\partial x} \left\{ h^{n+2} \frac{1}{n} \frac{\partial p}{\partial x} \right\}
\]
\[
+ \frac{\partial}{\partial y} \left\{ h^{n+2} \frac{\partial p}{\partial y} \right\}
\]
\[
= 12m |V_x|^{n-1} \left[ \frac{\partial}{\partial x} \left( u_{1x} h + \frac{V_x h}{2} \right) \right] + 12m |V_x|^{n-1} \frac{\partial h}{\partial t},
\]
(29)

If \( s_x = 1, s_y = 0, n = 1, \) and \( b_1 = b_2 = b, \) then we have
\[
\frac{\partial}{\partial x} \left\{ h^3 \left( 1 + 6b \right) \frac{1}{h} \frac{\partial p}{\partial x} \right\}
\]
\[
+ \frac{\partial}{\partial y} \left\{ h^3 \left( 1 + 6b \right) \frac{\partial p}{\partial y} \right\}
\]
\[
= 12 \mu \left[ \frac{\partial}{\partial x} \left( u_{1x} h + \frac{V_x h}{2} \right) \right] + 12 \mu \frac{\partial h}{\partial t},
\]
(30)

3. 1-D problem

Now, we consider the 1-D wedge problem \( (b = h_0 + (x/l)m h_0) \) for the case of \( s_x = 1, s_y = 0, u_{1x} = u_0, \) and \( u_{2x} = 0, \) then Eq. (29) becomes
\[
\frac{\partial}{\partial x} \left\{ h^3 (h + b_1 + b_2)^{n-1} \left[ 1 + 3 \frac{b_1 + b_2}{h} - \frac{3(b_1 - b_2)^2}{h(h + b_1 + b_2)} \right] \frac{\partial p}{\partial x} \right\}
\]
\[
= 12m (V_x^2)^{1/2} \left[ \frac{\partial}{\partial x} \left( u_{1x} h + \frac{V_x h}{2} \right) \right],
\]
(31)

The above equation is non-dimensionalized by using \( X = x/l, \ H = h/h_0 = 1 + m^2 X, \ P = \frac{p}{\eta_0 \rho_0}, \ V_x = -u_0, \ \eta_0 = m \frac{\eta_0}{h_0} \), \( n = 1 \), \( b = \frac{b}{h_0} \). We have
\[
\frac{\partial}{\partial X} \left\{ \frac{G_1}{n} \frac{\partial P}{\partial X} \right\} = \frac{\partial G_1}{\partial X},
\]
(32)

where
\[
G_1 = 12 \left( H - \frac{H + 2B_1}{2H + B_1 + B_2} \right)
\]
(33)

and
\[
G_3 = H^3 (H + B_1 + B_2)^{n-1}
\]
\[
\times \left[ 1 + 3 \frac{B_1 + B_2}{H} - \frac{3(B_1 - B_2)^2}{H(H + B_1 + B_2)} \right].
\]
(34)

Eq. (32) is discretized by the central difference technique (Fig. 1), i.e.,
\[
\alpha P_{i+1} + \delta' P_i + \beta' P_{i-1} = c',
\]
(35)

where
\[
\alpha' = \frac{(G_3)_R}{k \Delta X(i)}, \quad \beta' = \frac{(G_3)_L}{k \Delta X(i-1)},
\]
\[
c' = (G_3)_R - (G_3)_L, \quad \delta' = -\delta' - \beta'.
\]

Fig. 1. Grid points.
With the boundary conditions: \( P_1 = 0 \) (at \( X = 0 \)), \( P_{n+1} = 0 \) (at \( X = 1 \)), Eq. (35) is solved by the tri-diagonal matrix inverse.

4. Velocity distributions

The velocity distributions for \( s_x = 1, s_y = 0 \) are

\[
u = u_{1x} + \frac{z + b_1}{h + b_1 + b_2} (u_{2x} - u_{1x})
\]

\[+rac{1}{\mu_0 n \partial X} \left[ \frac{z^2}{2} \frac{h (h + 2b_2)(z + b_1)}{h + b_1 + b_2} \right] = u_C + u_P \tag{36}
\]

if \( u_{1x} = u_b, \) and \( u_{2x} = 0, \) then \( V_x = -u_b, \) and thus

\[
u = u_{b} + \frac{z + b_1}{h + b_1 + b_2} (-u_b)
\]

\[+rac{1}{\mu_0 n \partial X} \left[ \frac{z^2}{2} \frac{h (h + 2b_2)(z + b_1)}{h + b_1 + b_2} \right] \tag{37}
\]

where

\[
\mu_0 = \eta_0 \left( \frac{1}{H + B_1 + B_2} \right)^{n-1}, \quad \eta_0 = m \left( \frac{u_b}{\eta_0} \right)^{n-1}. \tag{38}
\]

Eq. (37) is non-dimensionalized by \( u_b, \) i.e. \( U = U_P + U_C \) where

\[
U_C = \frac{u_C}{u_b} = 1 - \frac{Z + B_1}{H + B_1 + B_2}, \tag{39}
\]

\[
U_P = \frac{u_P}{u_b} = \frac{1}{n \partial X} \left[ \frac{Z^2}{2} + \frac{H (H + 2B_2)(Z + B_1)}{H + B_1 + B_2} \right]
\]

\[\times (H + B_1 + B_2)^{n-1}. \tag{40}\]

5. Results and discussions

In the 1-D wedge problem, the upper plane is fixed and inclined, and the lower plane is moving with \( U = 1. \) From Eq. (32), the pressure increases as the Poiseuille term \( (1/n G_3) \) decreases, or as the Couette term \( (G_1) \) increases. As shown in Fig. 2, the Couette flow for Newtonian fluid are plotted for four slip conditions, i.e. \( (B_1, B_2) = (0, 0), (0, 1, 0.1), (0.1, 0.2), \) and \( (0.2, 0.1). \) The slip velocities on the boundaries are the same if \( B_1 = B_2. \) The velocity profiles are parallel for the cases of \( (B_1, B_2) = (0.1, 0.2) \) and \( (0.2, 0.1). \) The Couette term for \( (B_1, B_2) = (0.1, 0.2) \) is greater than that for \( (B_1, B_2) = (0.2, 0.1). \) As shown in Fig. 3, the Poiseuille flow for Newtonian fluid are plotted for four slip conditions, i.e. \( (B_1, B_2) = (0, 0), (0.1, 0), (0.1, 0.1), (0.1, 0.2), \) and \( (0.2, 0.1) \). The velocity profiles are not symmetric about the middle plane \( (Z = 0.5) \). For the cases of \( (B_1, B_2) = (0.1, 0.2) \) and \( (B_1, B_2) = (0.2, 0.1), \) the velocity profiles are anti-symmetric about the middle plane \( (Z = 0.5) \). The Poiseuille terms are the same for these two cases. Due to the contribution of Couette terms, the pressure distributions for \( (B_1, B_2) = (0.1, 0.2) \) is greater than that for \( (B_1, B_2) = (0.2, 0.1), \) and thus we have the load capacity: \( W(0.1, 0.2) > W(0.2, 0.1). \)

In Fig. 4, the Poiseuille flow are plotted for \( n = 0.9, 1.0, 1.1. \) The Poiseuille term follows: shear thinning \( (n < 1) \) > Newtonian \( (n = 1) \) > shear thickening \( (n > 1). \) Also, the larger the slip length is, the larger the Poiseuille flow is.
Thus, the Poiseuille term $U_p(B_1, B_2)$ follows: $U_p(0.3, 0.1) > U_p(0.3, 0.3) > U_p(0.3, 0.5)$. The slip lengths on moving boundary are the same ($B_1 = 0.3$), and the load capacities for larger $B_2$ is smaller due to the less resistance. Larger slip length results in smaller resistance on Poiseuille flow, and thus results in smaller load capacities.

The load capacities for various flow indices ($n$) are plotted as shown in Figs. 5 and 6. The load capacities follow: $W(n > 1) > W(n = 1) > W(n < 1)$, $W(0, B_2) > W(0.3, B_2)$, and $W(B_2, 0) > W(B_2, 0.3)$. In Fig. 7, the load capacities are plotted for various flow indices and slip lengths (partially wetted). Three types of slip pairs are considered, i.e. $(B_1, B_2) = (0.3, B)$, $(B, 0.3)$, and $(BB)$. For $B > 0.3$, we have $W(0.3, B) > W(B, 0.3) > W(B, B)$. For $B < 0.3$, we have $W(0.3, B) < W(B, 0.3) < W(B, B)$. In Fig. 8, one lubricating surface follows the no-slip conditions. We have $W(0, B) > W(B, 0) > W(B, B)$. From Figs. 7 and 8, the variation of load capacity with respect to the slip length, $\partial W/\partial B$, is more sensitive for $(B, B)$, i.e.

$$\frac{\partial W}{\partial B}(B, B) > \frac{\partial W}{\partial B}(B, 0.3) > \frac{\partial W}{\partial B}(0.3, B)$$

and

$$\frac{\partial W}{\partial B}(B, B) > \frac{\partial W}{\partial B}(B, 0) > \frac{\partial W}{\partial B}(0, B).$$

From the above discussion, the effects of slip and flow rheology affect the lubrication performance significantly. The existence of surface slip or flow rheology of shear thinning resists the flow between lubricating surfaces less as compared to that for no-slip or shear thickening lubricant. Therefore, they result in lower load capacities. In addition, the combination of these two effects (slip and rheology) will result in the same result as that calculated from Newtonian fluid with no-slip boundary conditions.

6. Conclusion

In this paper, the effects of slip velocity and flow rheology on the hydrodynamic pressure can be discussed
from the derived extended Reynolds equation. The results show that smaller slip length on lubricating boundaries result in larger Couette term and smaller Poiseuille term, and thus result in larger hydrodynamic pressure. The load capacities follow: (1) effect of rheology: \( W(n<1) < W(n=1) < W(n>1) \), (2) effect of slip lengths: \( W(0,0) > W(B_c,B) > W(B_c,B_c) > W(B,B) \) for \( B>B_c \), and \( W(0,0) > W(B,B) > W(B_c,B) \) for \( B>B_c \). (3) sensitivity to slip length:

\[
\frac{\partial W}{\partial B}(B,B) > \frac{\partial W}{\partial B}(B,B_c) > \frac{\partial W}{\partial B}(B_c,B).
\]

Acknowledgment

This research was supported by NSC at ROC, Contract No NSC 93-2212-E-151-009.

References


