Types of Follower Motions

Parabolic Motion (Return)

For the 1st half, \( 0 \leq \theta^* \leq \beta / 2 \)

B. C.: at \( \theta^* = 0, \quad s^* = L, \quad \dot{s}^* = 0; \)

\( \text{at } \theta^* = \beta / 2, \quad s^* = L / 2. \)

Let \( s^* = C_0 + C_1 \theta^* + C_2 (\theta^*)^2 \)

\( \Rightarrow C_0 = L, \quad C_1 = 0, \quad C_2 = -2L / \beta^2 \)

\( \Rightarrow s^* = L \left[ 1 - 2 \left( \frac{\theta^*}{\beta} \right)^2 \right] \)

For the 2nd half, \( \beta / 2 \leq \theta^* \leq \beta, \)

B. C.: at \( \theta^* = \beta / 2, \quad s^* = L / 2, \)

\( \text{at } \theta^* = \beta, \quad s^* = \dot{s}^* = 0. \)

\( \Rightarrow s^* = 2L \left[ 1 - \left( \frac{\theta^*}{\beta} \right)^2 \right] \)

\( \Rightarrow s^* \text{ (return)} = L - s^* \text{ (rise)} \)

\( s^* \text{ (rise)} = 2L \left( \frac{\theta^*}{\beta} \right)^2 \)

\( s = L \left[ 1 - 2 \left( 1 - \frac{\theta^*}{\beta} \right)^2 \right] \)
Types of Follower Motions

Harmonic Motion

- Harmonic motion incorporates a portion of a sine wave. (271-3-1)

Let \( s^* = C_0 + C_1 \cos C_2 \theta^* \)

B. C.: at \( \theta^* = 0 \), \( s^* = \dot{s}^* = 0 \);

at \( \theta^* = \beta \), \( s^* = L \).

\[ \Rightarrow s^* = \frac{L}{2} \left( 1 - \cos \frac{\pi \theta^*}{\beta} \right) \]

A return of \( L \) is required in \( \beta \) radians

B. C.: at \( \theta^* = 0 \), \( s^* = L \), \( \dot{s}^* = 0 \);

at \( \theta^* = \beta \), \( s^* = 0 \).

\[ \Rightarrow s^* = \frac{L}{2} \left( 1 + \cos \frac{\pi \theta^*}{\beta} \right) \]
Types of Follower Motions

Cycloidal (擺線) Motion

\[ L = 2\pi r \quad \Rightarrow \quad r = \frac{L}{2\pi} \]

\[ \theta = \frac{2\pi \phi}{\beta} \quad \Rightarrow \quad \phi = \frac{\beta}{2\pi} \]

\[ \phi = \frac{L - 2\pi r}{\beta} = \frac{L}{2\pi} \sin \left( \frac{2\pi \phi}{\beta} \right) \]

\[ s^* = \frac{\left( \frac{\theta}{\beta} - \frac{1}{2\pi} \right)}{2\pi} \]

A rise of \( L \) is required in \( \beta \) radians.
Types of Follower Motions

Cycloidal (擺線) Motion

A rise of \( L \) is required in \( \beta \) radians

\[
s^* = L \left( \frac{\theta^*}{\beta} - \frac{1}{2\pi} \sin \frac{2\pi \theta^*}{\beta} \right)
\]

\[
\Rightarrow s^* = \frac{L\omega}{\beta} \left( 1 - \cos \frac{2\pi \theta^*}{\beta} \right)
\]

\[
\ddot{s}^* = \frac{2L\pi \omega^2}{\beta^2} \sin \frac{2\pi \theta^*}{\beta}
\]

- Acceleration is zero at the start and finish, and the jerk remains finite throughout the entire motion. (271-5-3)
Comparison of Follower Motions

In general, it is required that the displacement, velocity, and acceleration of the follower motion are continuous and the jerk is finite.

Among the four motions, only the cycloidal motion can be used to joint a dwell motion and satisfy the above requirements.